

الافكار المركزية

1 – instantaneous power

2- Active power

3 – Reactive power

4 – Apparent power

5 – Complex power

6 – Power Triangle

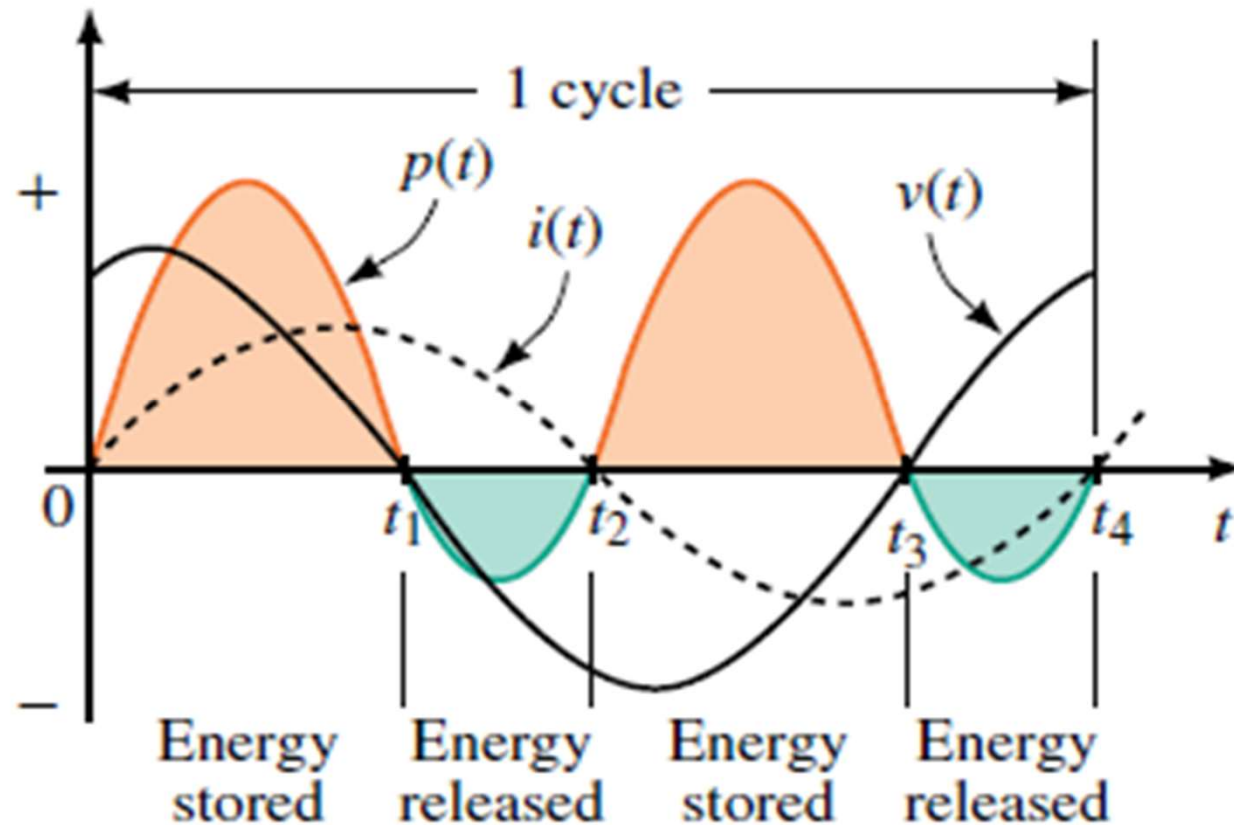
7 – Power Factor

8 – Power Factor Correction

9 - Examples

instantaneous power

At any instant , the power is equal to the product of voltages times current

$$p = vi \quad (\text{watts})$$


Active power (p)

mean Is the average value of the instantaneous power ,

The terms real power , active power , average power

Means same things

$$p = VI \cos \theta$$

$$P = VI = \frac{V_m I_m}{2} = I^2 R = \frac{V^2}{R}$$

(watts, W)

Reactive power (Q)

This power happen if the load contains reactive element L, C

$$Q = VI \sin \theta$$

(volt-ampere reactive, VAR)

where θ is the phase angle between V and I .

The Reactive power to the pure inductor can be written as below

where θ is the phase angle between V and I .

For the inductor,

$$Q_L = VI \quad (\text{VAR})$$

or, since $V = IX_L$ or $I = V/X_L$,

$$Q_L = I^2 X_L \quad (\text{VAR})$$

or

$$Q_L = \frac{V^2}{X_L} \quad (\text{VAR})$$

The reactive power to the capacitor can be written as below

$$Q_C = VI \quad (\text{VAR})$$

$$Q_C = I^2 X_C \quad (\text{VAR})$$

$$Q_C = \frac{V^2}{X_C} \quad (\text{VAR})$$

Apparent Power (S)

If the load contains both resistance and reactance then the product of voltage (v) and current (I) represents neither real power nor reactive power , it is called apparent power

$$S = VI \quad (\text{volt-amperes, VA})$$

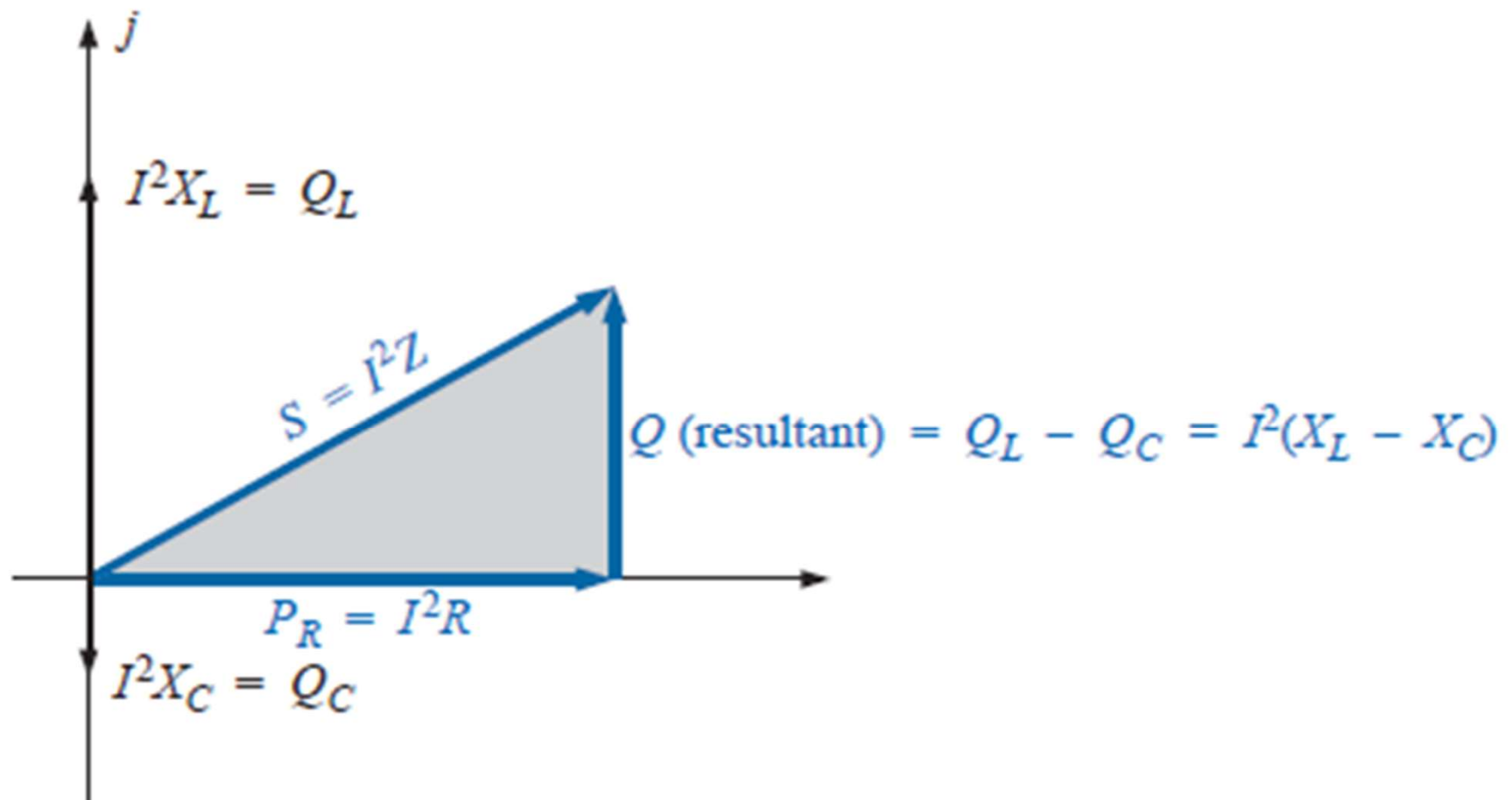
or, since $V = IZ$ and $I = \frac{V}{Z}$

then $S = I^2Z$ (VA)

and $S = \frac{V^2}{Z}$ (VA)

complex power

$$S = VI^*$$

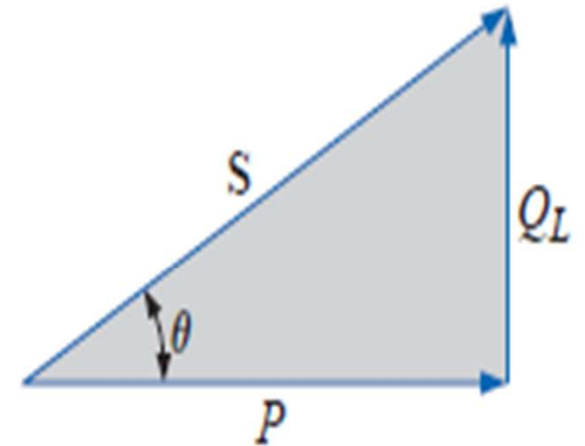


Power triangle

$$P = P \angle 0^\circ \quad Q_L = Q_L \angle 90^\circ \quad Q_C = Q_C \angle -90^\circ$$

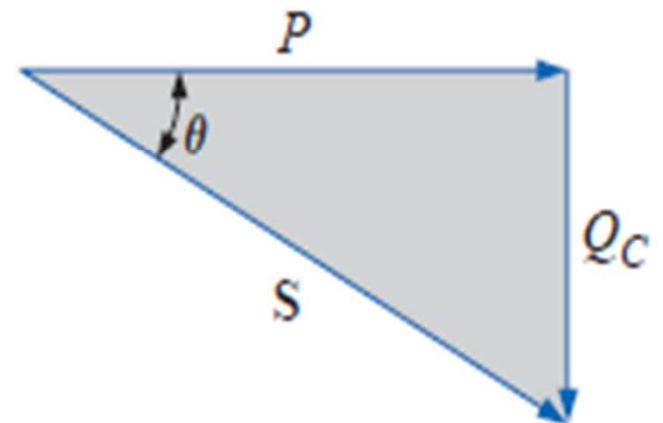
For an inductive load, the *phasor power* S , as it is often called, is defined by

$$S = P + jQ_L$$



For a capacitive load, the phasor power S is defined by

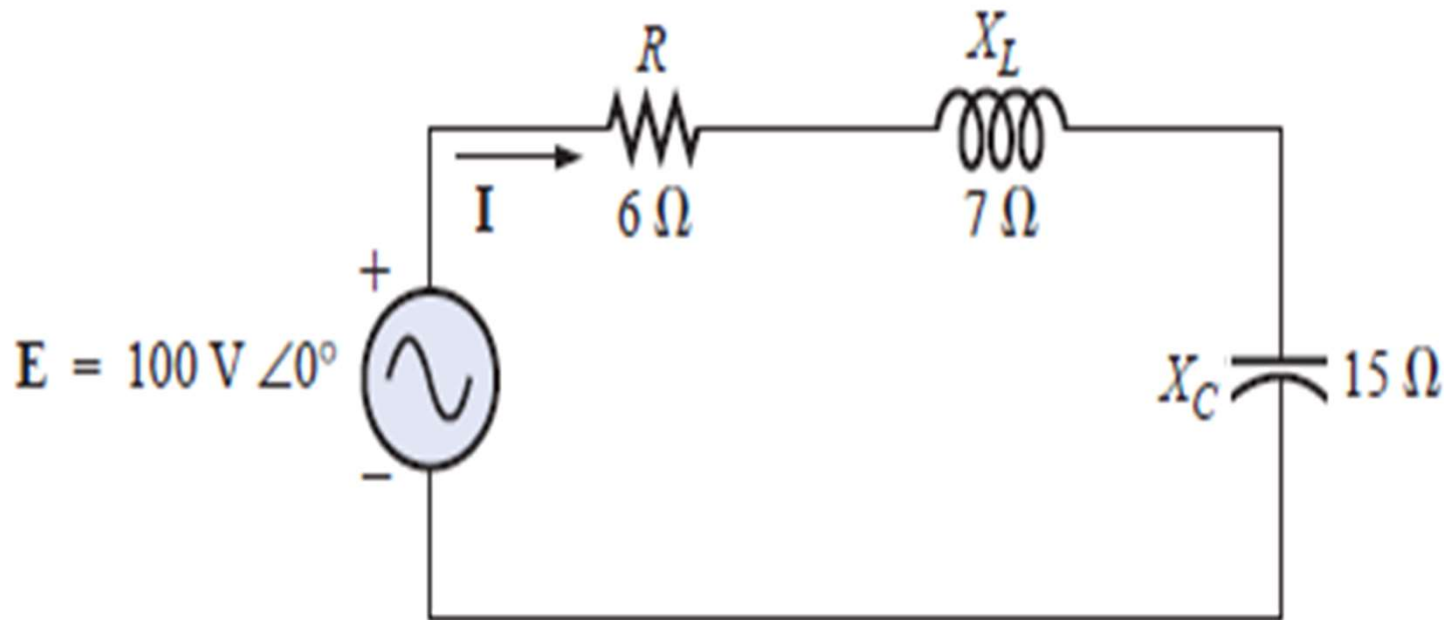
$$S = P - jQ_C$$



THE TOTAL P , Q , AND S

1. Find the real power and reactive power for each branch of the circuit.
2. The total real power of the system (P_T) is then the sum of the average power delivered to each branch.
3. The total reactive power (Q_T) is the difference between the reactive power of the inductive loads and that of the capacitive loads.
4. The total apparent power is $S_T = \sqrt{P_T^2 + Q_T^2}$.
5. The total power factor is P_T/S_T .

Example : for the circuit shown find the total power ,the total reactive power , apparent power , draw power triangle



Solutions:

$$\text{a. } \mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100 \text{ V } \angle 0^\circ}{6 \Omega + j7 \Omega - j15 \Omega} = \frac{100 \text{ V } \angle 0^\circ}{10 \Omega \angle -53.13^\circ}$$
$$= 10 \text{ A } \angle 53.13^\circ$$

$$\mathbf{V}_R = (10 \text{ A } \angle 53.13^\circ)(6 \Omega \angle 0^\circ) = 60 \text{ V } \angle 53.13^\circ$$

$$\mathbf{V}_L = (10 \text{ A } \angle 53.13^\circ)(7 \Omega \angle 90^\circ) = 70 \text{ V } \angle 143.13^\circ$$

$$\mathbf{V}_C = (10 \text{ A } \angle 53.13^\circ)(15 \Omega \angle -90^\circ) = 150 \text{ V } \angle -36.87^\circ$$

$$\mathbf{P}_T = EI \cos \theta = (100 \text{ V})(10 \text{ A}) \cos 53.13^\circ = 600 \text{ W}$$

$$= I^2 R = (10 \text{ A})^2 (6 \Omega) = 600 \text{ W}$$

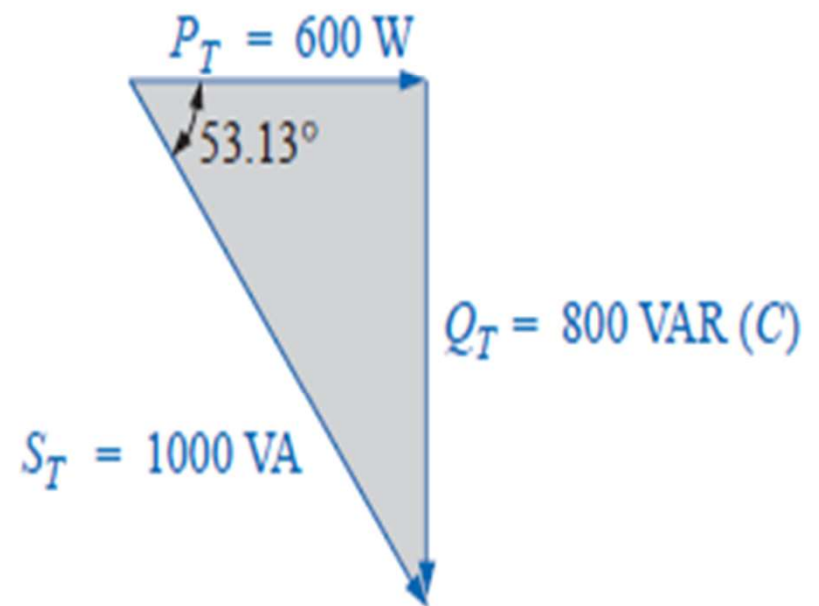
$$= \frac{V_R^2}{R} = \frac{(60 \text{ V})^2}{6} = 600 \text{ W}$$

$$\begin{aligned} S_T &= EI = (100 \text{ V})(10 \text{ A}) = 1000 \text{ VA} \\ &= I^2 Z_T = (10 \text{ A})^2 (10 \Omega) = 1000 \text{ VA} \\ &= \frac{E^2}{Z_T} = \frac{(100 \text{ V})^2}{10 \Omega} = 1000 \text{ VA} \end{aligned}$$

$$\begin{aligned} Q_T &= EI \sin \theta = (100 \text{ V})(10 \text{ A}) \sin 53.13^\circ = 800 \text{ VAR} \\ &= Q_C - Q_L \\ &= I^2 (X_C - X_L) = (10 \text{ A})^2 (15 \Omega - 7 \Omega) = 800 \text{ VAR} \end{aligned}$$

$$Q_T = \frac{V_C^2}{X_C} - \frac{V_L^2}{X_L} = \frac{(150 \text{ V})^2}{15 \Omega} - \frac{(70 \text{ V})^2}{7 \Omega}$$
$$= 1500 \text{ VAR} - 700 \text{ VAR} = 800 \text{ VAR}$$

$$F_p = \frac{P_T}{S_T} = \frac{600 \text{ W}}{1000 \text{ VA}} = 0.6 \text{ leading (C)}$$



Example: find complex power

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{10 \text{ V} \angle 0^\circ}{3 \Omega + j4 \Omega} = \frac{10 \text{ V} \angle 0^\circ}{5 \Omega \angle 53.13^\circ} = 2 \text{ A} \angle -53.13^\circ$$

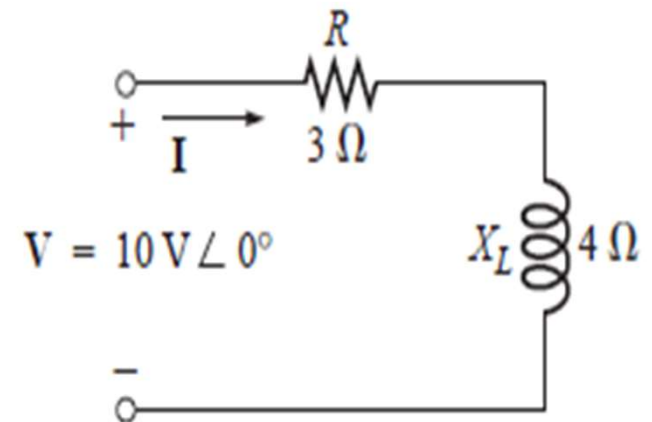
The real power (the term *real* being derived from the positive real axis of the complex plane) is

$$P = I^2 R = (2 \text{ A})^2 (3 \Omega) = 12 \text{ W}$$

and the reactive power is

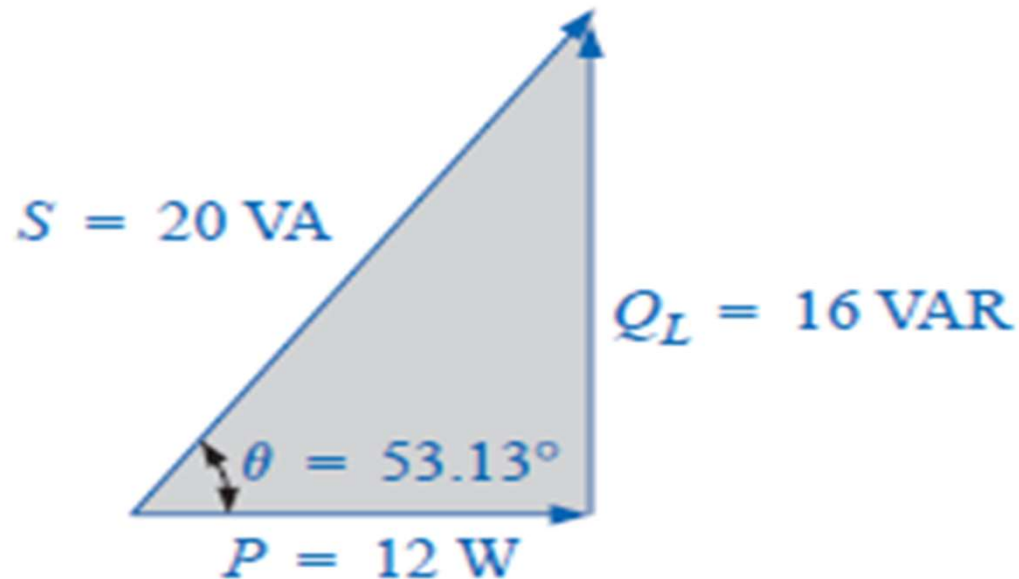
$$Q_L = I^2 X_L = (2 \text{ A})^2 (4 \Omega) = 16 \text{ VAR } (L)$$

with $\mathbf{S} = P + jQ_L = 12 \text{ W} + j16 \text{ VAR } (L) = 20 \text{ VA} \angle 53.13^\circ$



Complex power and power triangle

$$S = VI^* = (10 \text{ V} \angle 0^\circ)(2 \text{ A} \angle +53.13^\circ) = 20 \text{ VA} \angle 53.13^\circ$$



Example: find P , Q , S , P.f, I_t

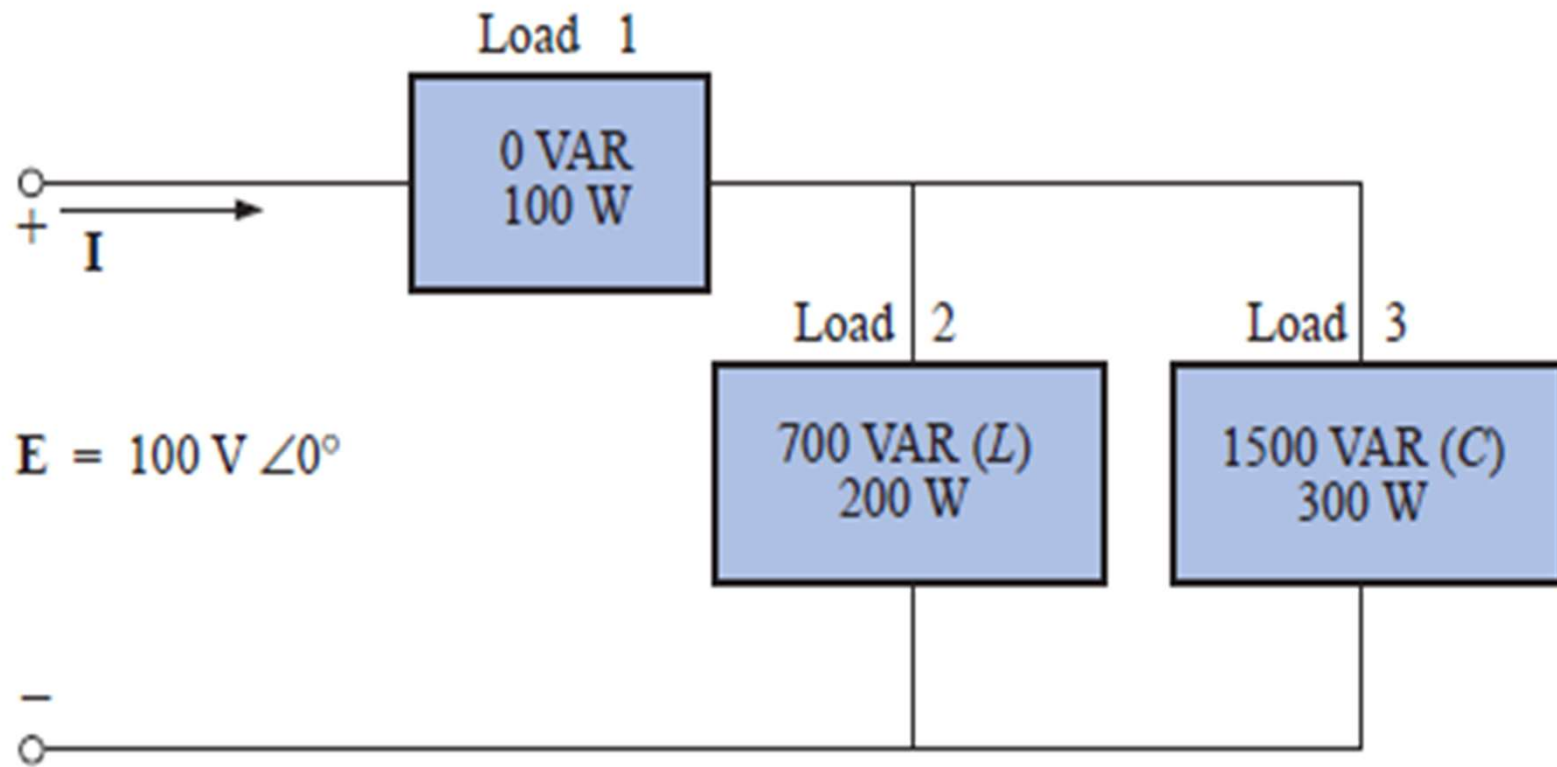


TABLE 10.1

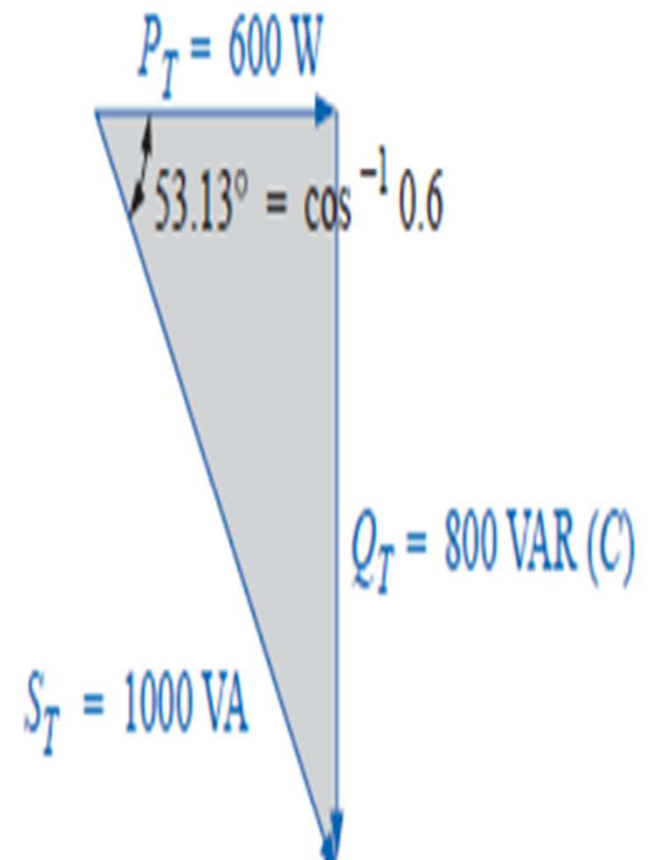
| Load | W | VAR | VA |
|------|------------------------|-------------------------------------|---|
| 1 | 100 | 0 | 100 |
| 2 | 200 | 700 (L) | $\sqrt{(200)^2 + (700)^2} = 728.0$ |
| 3 | 300 | 1500 (C) | $\sqrt{(300)^2 + (1500)^2} = 1529.71$ |
| | <u> </u> | <u> </u> | <u> </u> |
| | $P_T = 600$ | $Q_T = 800 (C)$ | $S_T = \sqrt{(600)^2 + (800)^2} = 1000$ |
| | Total power dissipated | Resultant reactive power of network | (Note that $S_T \neq$ sum of each branch: $1000 \neq 100 + 728 + 1529.71$) |

$$F_p = \frac{P_T}{S_T} = \frac{600 \text{ W}}{1000 \text{ VA}} = 0.6 \text{ leading (C)}$$

The power triangle is shown in Fig. 19.18.

Since $S_T = VI = 1000 \text{ VA}$, $I = 1000 \text{ VA}/100 \text{ V} = 10 \text{ A}$; and since θ of $\cos \theta = F_p$ is the angle between the input voltage and current:

$$I = 10 \text{ A} \angle +53.13^\circ$$



power factor

Power Factor (P.F)

عامل القدرة

It is define as :-

1-cosine of phase angle.

2-The ratio of $\frac{P}{S}$.

3-The ratio of $\frac{R}{Z}$.

Inductive circuit has Lagging power factor
And capacitive circuits has Leading power factor

Example:- For the circuit shown Find P, Q, S, P.F?

$$Z = 6 + j(7 - 15) = 6 - j8$$

$$Z = \sqrt{6^2 + 8^2} = 10 \Omega$$

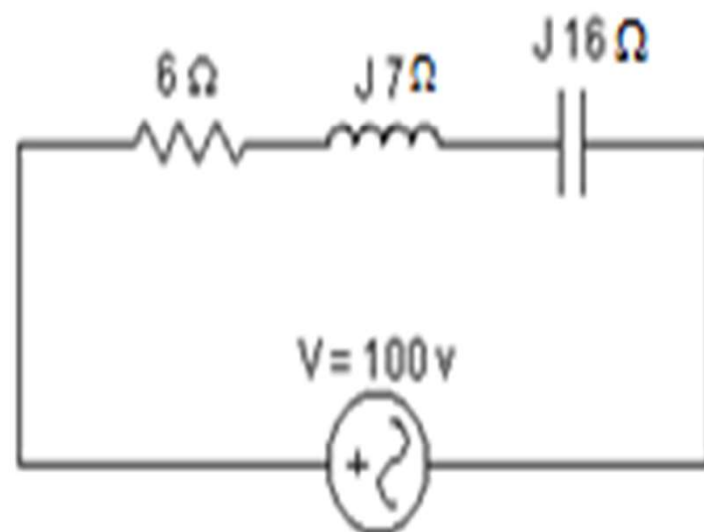
$$I = \frac{V}{Z} = \frac{100}{10} = 10 \text{ A}$$

$$P = I^2 \times R = 10^2 \times 6 = 600 \text{ W}$$

$$Q = I^2 \times X = 10^2 \times 8 = 800 \text{ W}$$

$$S = V \times I = 10 \times 100 = 1000 \text{ W}$$

$$P.f = \frac{P}{S} = \frac{600}{1000} = 0.6 \quad (\text{Lead}).$$



Example:- For the circuit Find Total P , Q , S , P.F?

$$Z_1 = 3 - j4 = 5 \angle 53^\circ$$

$$I_1 = \frac{V}{Z_1} = \frac{60}{5} = 12 \text{ A}$$

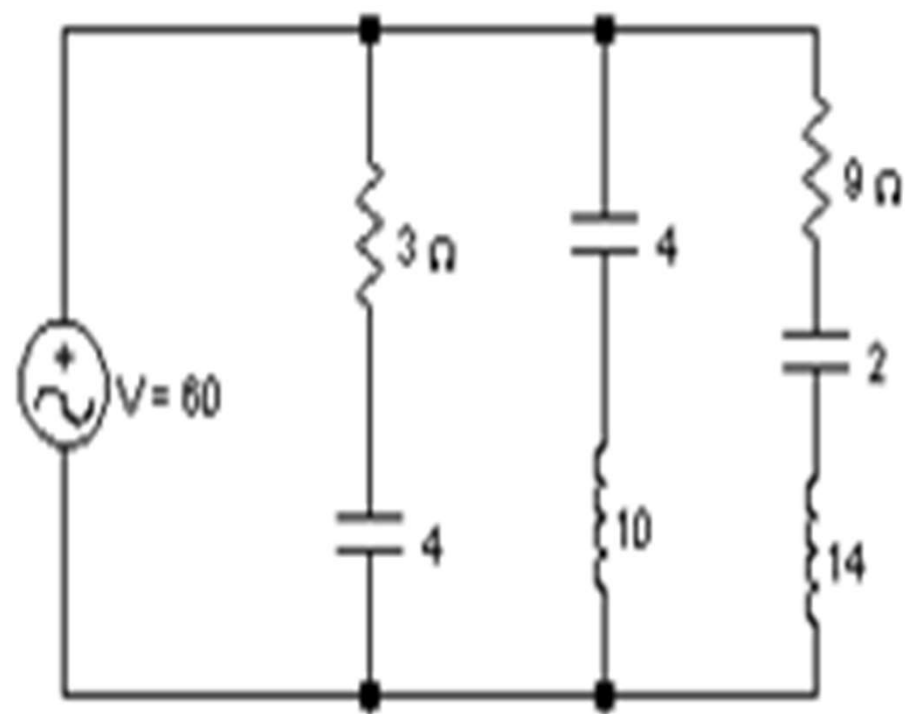
$$P_1 = I_1^2 \times R_1 = 12^2 \times 3 = 432 \text{ W}$$

$$Q_1 = I_1^2 \times X_1 = 12^2 \times 4 = 576 \text{ VAR}$$

$$Z_2 = j10 - j4 = 0 + j6 = 6 \angle 90^\circ$$

$$I_2 = \frac{60}{6} = 10 \text{ A}$$

$$P_2 = 0 \quad , \quad Q_2 = I^2 X = 10^2 \times 6 = 600 \text{ VAR (Inductive)}$$



$$Z_3 = 9 + j14 - j2 = 9 + j12 = 15 \angle 53^\circ \quad , \quad I_3 = \frac{60}{15} = 4 \text{ A}$$

$$P_3 = I_3^2 \times R_3 = 4^2 \times 9 = 144 \text{ W} \quad , \quad Q_3 = 4^2 \times 12 = 192 \text{ VAR (Inductive)}$$

$$P_T = P_1 + P_2 + P_3 = 432 + 0 + 144 = 576 \text{ W}$$

$$Q_T = Q_1 + Q_2 + Q_3 = -576 + 600 + 192 = 216 \text{ VAR}$$

$$S = \sqrt{P^2 + Q^2} = 615 \text{ W} \quad , \quad \text{Pf} = \frac{P}{S} = 0.93 \text{ Lag.}$$

power factor correction

The following equipment is generally used to improve or correct the power – factor :

- 1 – synchronance motor when they are over – excited
- 2 – static capacitor

When p.f is Low the current required for given power is very high and K V A IS also increased

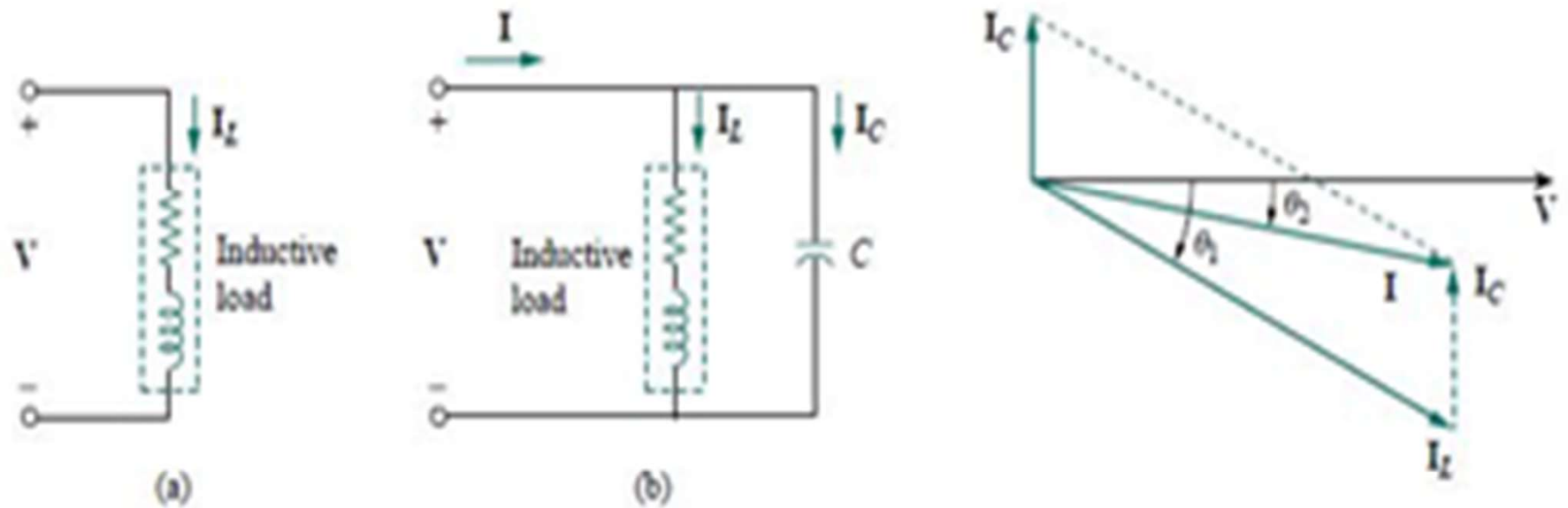
$$KVA = \frac{KW}{P.F} , I = \frac{KVA}{V} , I \propto KVA$$

The process of increasing the power – factor with out altering the voltage and current of the original Load is known as power – factor correction

Also cancelling some or all of the reactive component of power by adding reactance of the opposite type to the circuit, this is referred to as power –factor correction

Since most Loads are inductive as shown in fig- a

The p.f is improved or correct by connecting acapacitor in parallel with Load as shown in fig- b



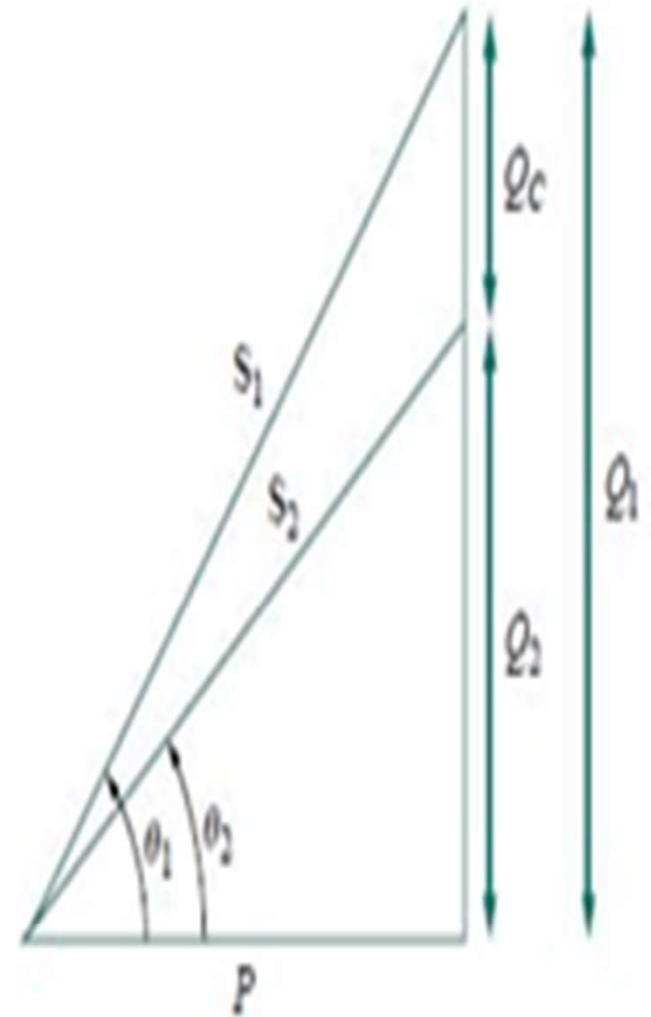
Consider the power triangle in fig-

If the original inductive Load has

Apparent power S_1

$$P = S_1 \cos\theta_1 \quad , \quad S_1 = \frac{P}{\cos\theta_1}$$

$$Q_1 = S_1 \sin\theta_1 = P \tan\theta_1$$



If we want to increase p.f from $\cos \theta_1$ to $\cos \theta_2$ with out altering the real power (p)

$$Q_2 = P \tan \theta_2 = S_2 \sin \theta_2 \quad , \quad S_2 = \frac{p}{\cos \theta_2} \quad ,$$

The reduction in the reactive power is caused by the shunt capacitor that is

$$Q_c = Q_1 - Q_2 = P \tan \theta_1 - p \tan \theta_2 \quad ,$$

$$Q_c = \frac{V^2}{X_c} = W C V^2 \quad , \quad C = \frac{Q_c}{W V^2} \quad ,$$

Example :

A Load consumed (4 kw) at lagging p – factor (0.8)

When connected to (120 v) , 60 Hz

Find the value of capacitor to raise p.f to (0.95)

Solution:

$$\cos\theta_1 = 0.8 \quad , \quad \theta_1 = \cos^{-1}(0.8) = 37^\circ \quad ,$$

$$S_1 = \frac{P}{\cos\theta_1} = \frac{4000}{0.8} = 5000 \text{ VA}$$

$$Q_1 = S_1 \sin\theta_1 = 5000 \sin 37 = 3000 \text{ VAR}$$

When the p.f is raised to 0.95

$$\cos\theta_2 = 0.95 \quad , \quad \theta_2 = \cos^{-1}(0.95) = 18.19^\circ \quad ,$$

$$S_2 = \frac{P}{\cos\theta_2} = \frac{4000}{0.95} = 4210.5 \text{ VA}$$

$$Q_2 = S_2 \sin\theta_2 = 4210.5 \sin 18.19 = 1314.4 \text{ VAR}$$

$$Q_c = Q_1 - Q_2 = 3000 - 1314.4 = 1685.6 \text{ VAR}$$

$$C = \frac{Q_c}{\omega V^2} = \frac{1685.6}{2\pi \cdot 60 \cdot 120^2} = 310.5 \mu\text{f}$$

Second method:

$$Q_1 = P \tan \theta_1 = 4000 * \tan 37 = 4000 * 0.75 = 3000 \text{ VAR}$$

$$Q_2 = P \tan \theta_2 = 4000 \tan 18.19 = 4000 * 0.33 = 1320 \text{ VAR}$$

$$Q_c = Q_1 - Q_2 = 3000 - 1320 = 1680 \text{ VAR}$$

$$C = \frac{Q_c}{\omega V^2} = \frac{1685.6}{2\pi * 60 * 120^2} = 310.5 \mu \text{ f}$$

Example :

The power factor of an industry drops below (0.85), the power of the component in it are as follow:

1- Lights $p_1 = 12 \text{ kw}$, $Q = 0$

2- Furnance $p_2 = 54 \text{ kw}$, $Q_2 = 72 \text{ KVAR}$

3- motor $P_m = 80 \text{ kw}$ at 0.8 lag.

a- Determine p_t , Q_t ,

b-) the value of capacitor required to bring p .f to

0.85 , c-) total current befor and after p.f correction

solution:

$$a) P_t = p_1 + p_2 + p_m = 12k + 54k + 80k = 146 \text{ kw}$$

$$Q_m = p \tan \theta_m = 80 \tan 37 = 60 \text{ KVAR}$$

$$Q_t = 0 + 54k + 60k = 132 \text{ KVAR} \quad S = 196.8 \text{ K L42}$$

$$b) - \theta_2 = \cos^{-1}(0.85) = 31.8^\circ$$

$$Q_2 = P \tan \theta_2 = 146k \tan 31.8 = 90.5k \text{ VAR}, S_2 = 171.8 \text{ K L31.8}$$

$$Q_c = Q_1 - Q_2 = 132 \text{ K} - 90.5 \text{ K} = 41.5 \text{ KVAR}$$

$$Q_c = \frac{V^2}{X_c}, X_c = \frac{V^2}{Q_c} = \frac{600^2}{41.5K} = 8.67 \Omega, c = \frac{1}{WX_c} = \frac{1}{2\pi f X_c} = 306 \text{ nf}$$

$$I_1 = \frac{S}{E} = \frac{196.8K}{600} = 328 \text{ A}, I_2 = \frac{S_2}{E} = \frac{171.8K}{600} = 286 \text{ A}$$

EXAMPLE 19.6

- a. A small industrial plant has a 10-kW heating load and a 20-kVA inductive load due to a bank of induction motors. The heating elements are considered purely resistive ($F_p = 1$), and the induction motors have a lagging power factor of 0.7. If the supply is 1000 V at 60 Hz, determine the capacitive element required to raise the power factor to 0.95.
- b. Compare the levels of current drawn from the supply.

Solutions:

a. For the induction motors,

$$S = VI = 20 \text{ kVA}$$

$$P = S \cos \theta = (20 \times 10^3 \text{ VA})(0.7) = 14 \times 10^3 \text{ W}$$

$$\theta = \cos^{-1} 0.7 \cong 45.6^\circ$$

and

$$Q_L = VI \sin \theta = (20 \times 10^3 \text{ VA})(0.714) = 14.28 \times 10^3 \text{ VAR (L)}$$

The power triangle for the total system appears in Fig. 19.28.

Note the addition of real powers and the resulting S_T :

$$S_T = \sqrt{(24 \text{ kW})^2 + (14.28 \text{ kVAR})^2} = 27.93 \text{ kVA}$$

with

$$I_T = \frac{S_T}{E} = \frac{27.93 \text{ kVA}}{1000 \text{ V}} = 27.93 \text{ A}$$

The desired power factor of 0.95 results in an angle between S and P of

$$\theta = \cos^{-1} 0.95 = 18.19^\circ$$

changing the power triangle to that of Fig. 19.29:

$$\begin{aligned} \text{with } \tan \theta &= \frac{Q'_L}{P_T} \rightarrow Q'_L = P_T \tan \theta = (24 \times 10^3 \text{ W})(\tan 18.19^\circ) \\ &= (24 \times 10^3 \text{ W})(0.329) = 7.9 \text{ kVAR (L)} \end{aligned}$$

The inductive reactive power must therefore be reduced by

$$Q_L - Q'_L = 14.28 \text{ kVAR (L)} - 7.9 \text{ kVAR (L)} = 6.38 \text{ kVAR (L)}$$

Therefore, $Q_C = 6.38 \text{ kVAR}$, and using

$$Q_C = \frac{E^2}{X_C}$$

we obtain

$$X_C = \frac{E^2}{Q_C} = \frac{(10^3 \text{ V})^2}{6.38 \times 10^3 \text{ VAR}} = 156.74 \Omega$$

$$\text{and } C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi)(60 \text{ Hz})(156.74 \Omega)} = 16.93 \mu\text{F}$$

$$\begin{aligned} \text{b. } S_T &= \sqrt{(24 \text{ kW})^2 + [7.9 \text{ kVAR (L)}]^2} \\ &= 25.27 \text{ kVA} \end{aligned}$$

$$I_T = \frac{S_T}{E} = \frac{25.27 \text{ kVA}}{1000 \text{ V}} = 25.27 \text{ A}$$

The new I_T is

$$I_T = 25.27 \text{ A} / 27.93 \text{ A} \quad (\text{original})$$

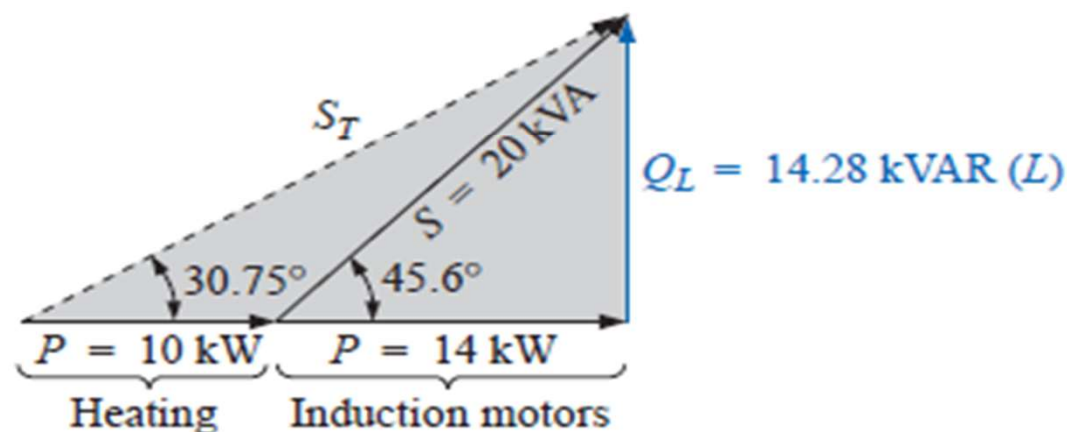


FIG. 19.28

Initial power triangle for the load of Example 19.6.

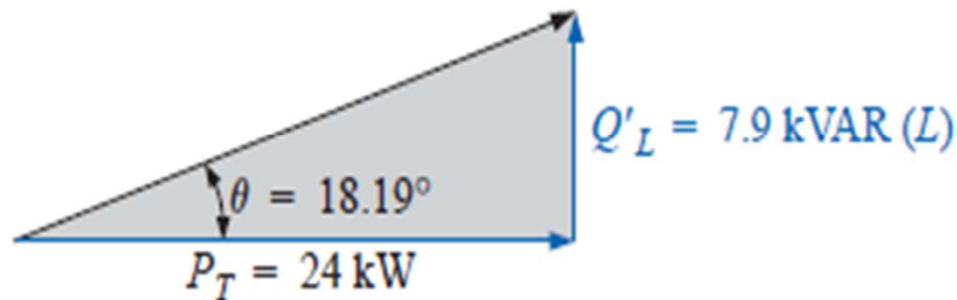
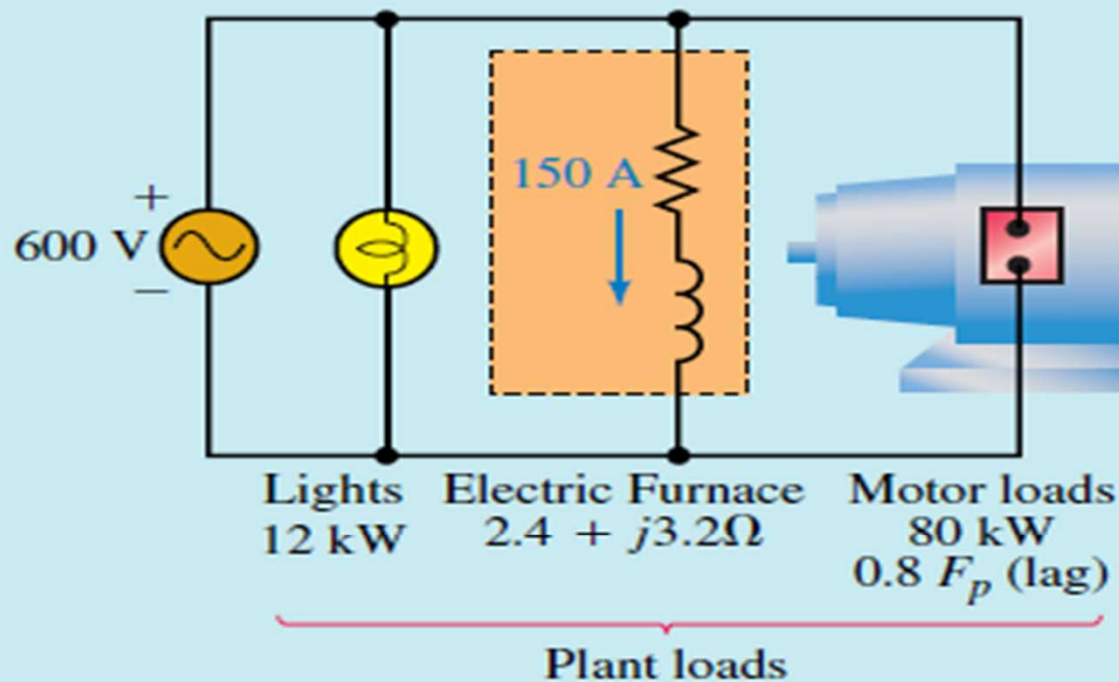


FIG. 19.29

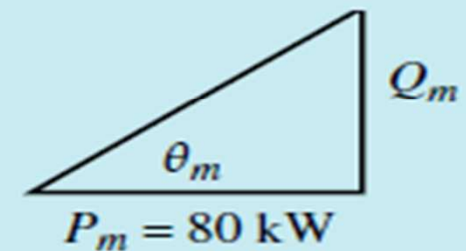
Power triangle for the load of Example 19.6 after raising the power factor to 0.95.

example: for circuit shown

- Determine P_T and Q_T .
- Determine what value of capacitance (in microfarads) is required to bring the power factor up to 0.85.
- Determine generator current before and after correction.

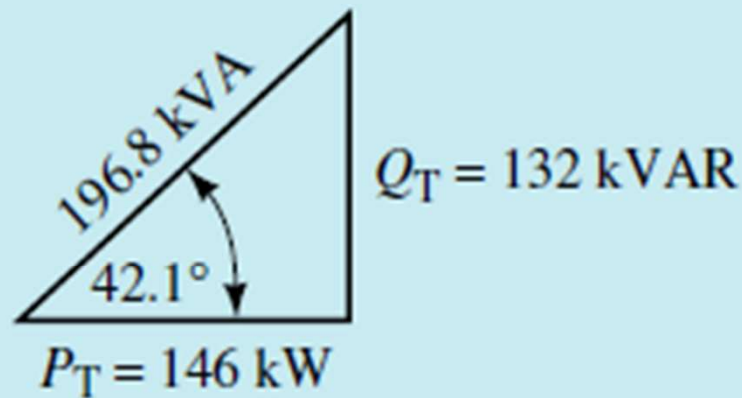


(a)

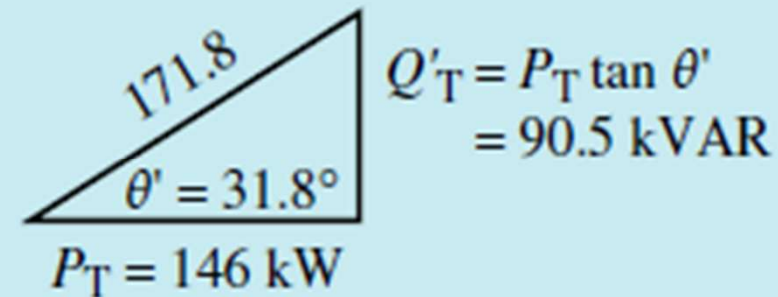


b) Power triangle for motor.

b. The power triangle for the plant is shown in Figure 17–21(a). However, we must correct the power factor to 0.85. Thus we need $\theta' = \cos^{-1}(0.85) = 31.8^\circ$, where θ' is the power factor angle of the corrected load as indicated in Figure 17–21(b). The maximum reactive power that we can tolerate is thus $Q'_T = P_T \tan \theta' = 146 \tan 31.8^\circ = 90.5 \text{ kVAR}$.



(a) Power triangle for the plant



(b) Power triangle after correction

FIGURE 17–21 Initial and final power triangles. Note that P_T does not change when we correct the power factor.

Now consider Figure 17-22. $Q'_T = Q_C + 132 \text{ kVAR}$, where $Q'_T = 90.5 \text{ kVAR}$. Therefore, $Q_C = -41.5 \text{ kVAR} = 41.5 \text{ kVAR (cap.)}$. But $Q_C = V^2/X_C$. Therefore, $X_C = V^2/Q_C = (600)^2/41.5 \text{ kVAR} = 8.67 \ \Omega$. But $X_C = 1/\omega C$. Thus a capacitor of

$$C = \frac{1}{\omega X_C} = \frac{1}{(2\pi)(60)(8.67)} = 306 \ \mu\text{F}$$

will provide the required correction.

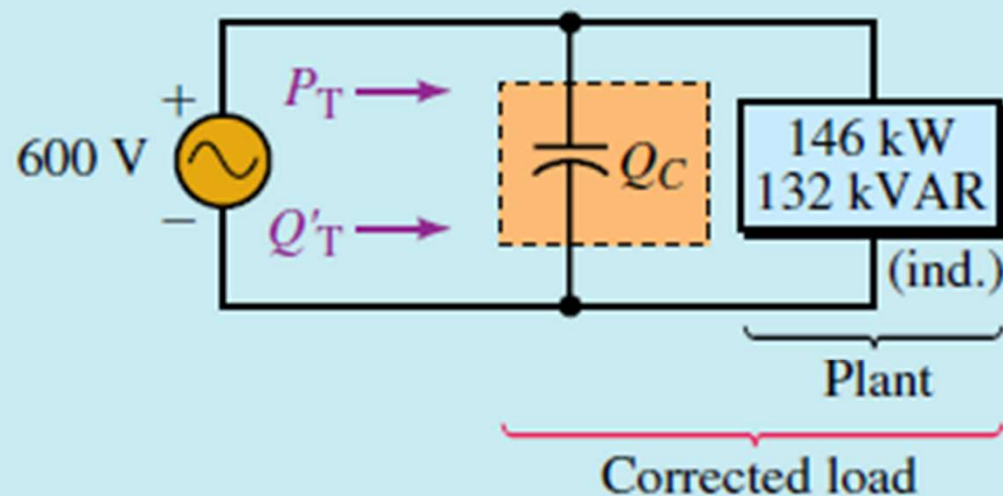


FIGURE 17-22

c. For the original circuit Figure 17–21(a), $S_T = 196.8 \text{ kVA}$. Thus,

$$I = \frac{S_T}{E} = \frac{196.8 \text{ kVA}}{600 \text{ V}} = 328 \text{ A}$$

For the corrected circuit 17–21(b), $S'_T = 171.8 \text{ kVA}$ and

$$I = \frac{171.8 \text{ kVA}}{600 \text{ V}} = 286 \text{ A}$$

Thus, power factor correction has dropped the current by 42 A.