Electrical Circuit

Kirchhoff's Laws

الافكار المركزية

1 - Defenitions

2 – kirchhoff,s current Law (KCL)

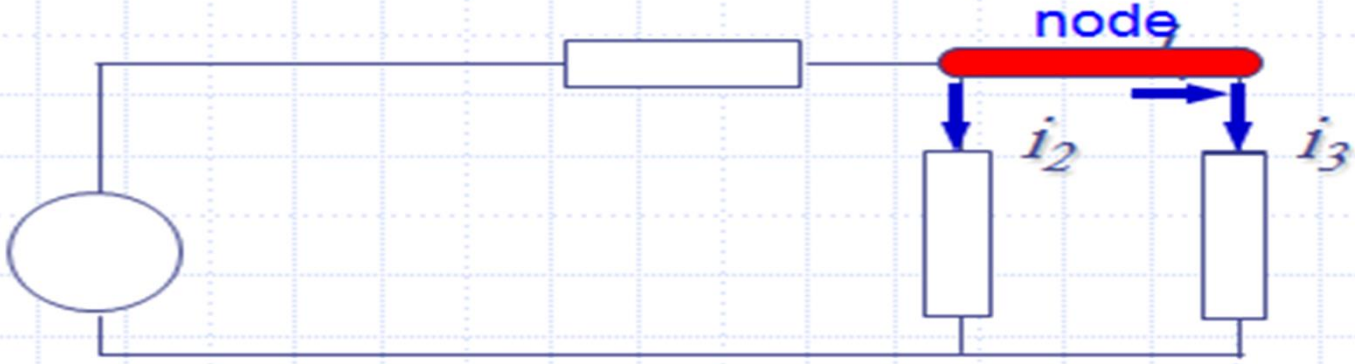
3 – Kirchhoff,s Voltage Law (KVL)

4 – Examples

5 – Home works

Kirchhoff's Current Law

- The sum of currents flowing **into** a node must be balanced by the sum of currents flowing **out** of the node.



i_1 flows **into** the node

i_2 flows **out** of the node

i_3 flows **out** of the node

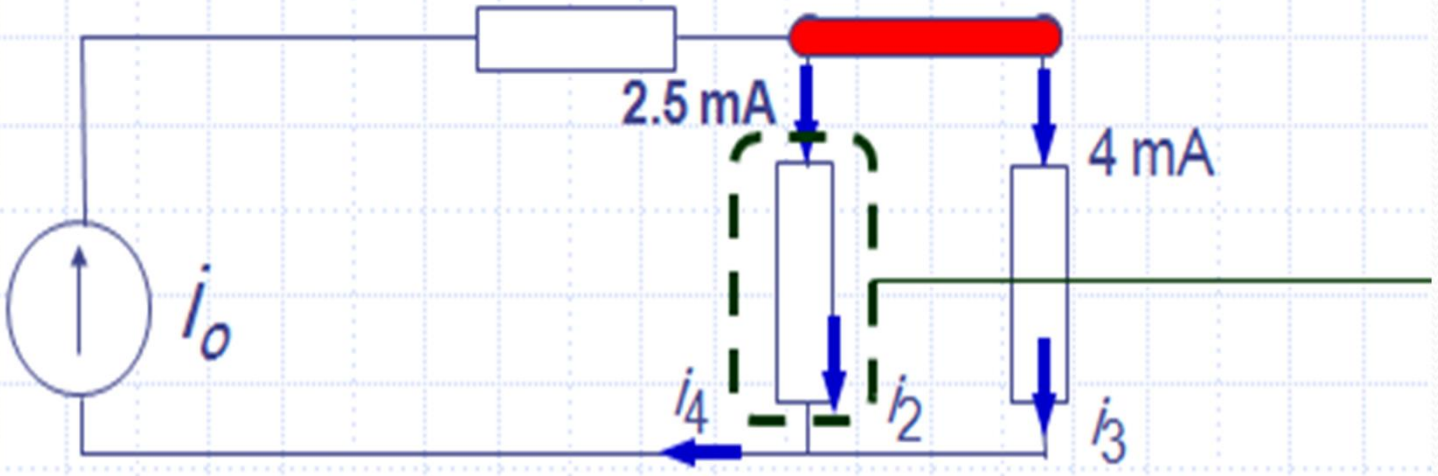
$$i_1 = i_2 + i_3$$

$$\sum i = 0$$

Example 1: Kirchhoff's Current Law:

Q: How much is the current I_o ?

A: $i_o = 2.5\ mA + 4\ mA = 6.5\ mA$

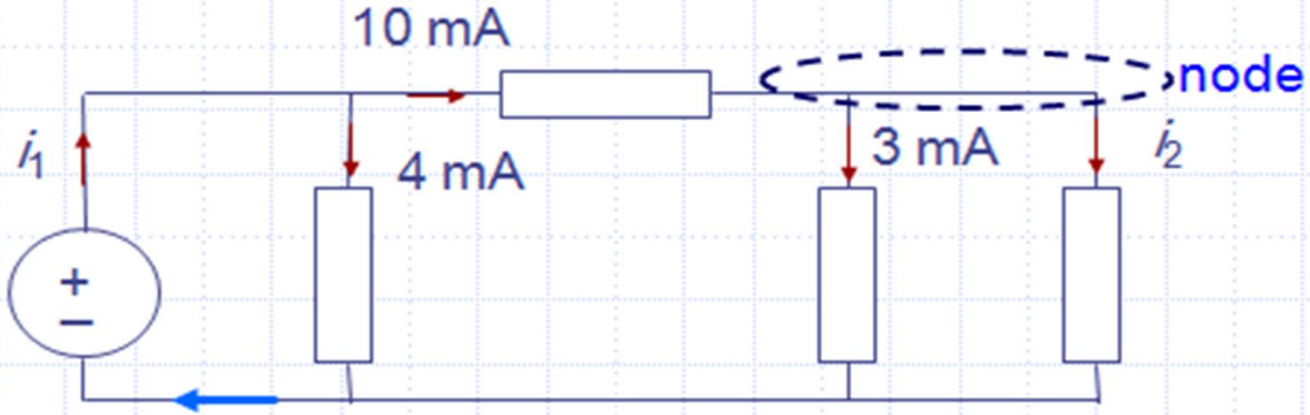


Example 2: Kirchhoff's Current Law:

Q: How much are the currents i_1 and i_2?

A: $i_2 = 10\ \text{mA} - 3\ \text{mA} = 7\ \text{mA}$

$i_1 = 10\ \text{mA} + 4\ \text{mA} = 14\ \text{mA}$



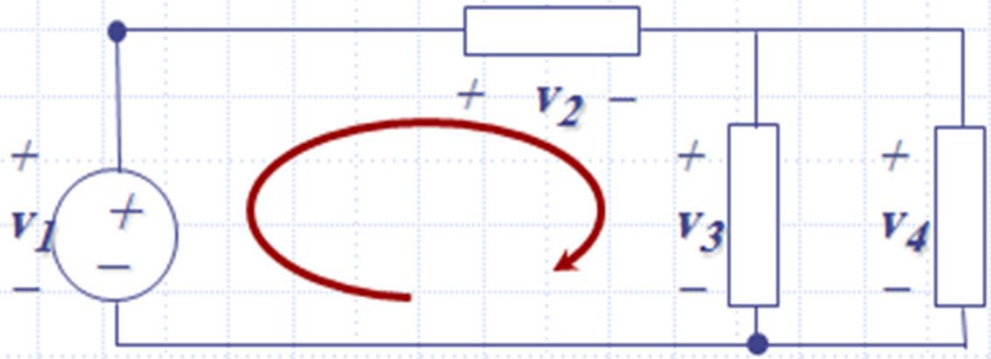
4 mA + 3 mA + 7 mA = 14 mA

Kirchhoff's Voltage Law:

$$v_1 = v_2 + v_3$$

- This equation can also be written in the following form:

$$-v_1 + v_2 + v_3 = 0$$



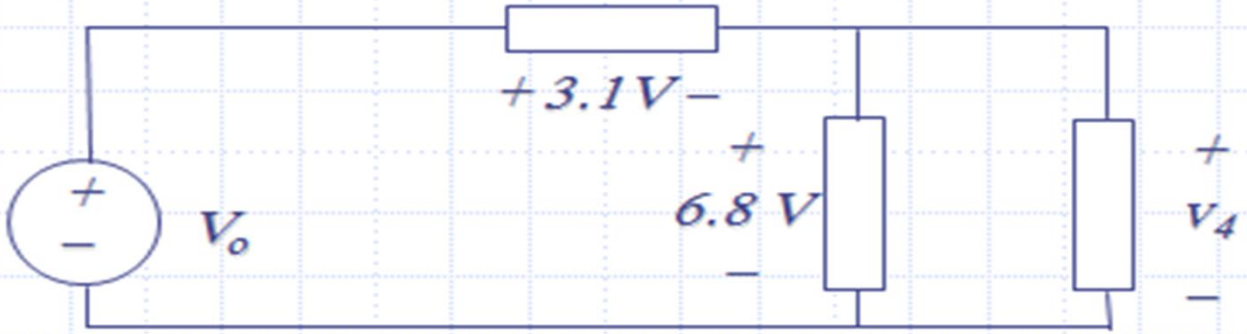
A formal statement of **Kirchhoff's Voltage Law**:

The sum of voltages around a **closed loop** is zero.

Example 1: Kirchhoff's Voltage Law:

Q: How much is the voltage V_o?

A: $V_o = 3.1\ V + 6.8\ V$



Q: How much is the voltage v_4?

A: $v_4 = 6.8\ V$

Example 2: Kirchhoff's Voltage Law:

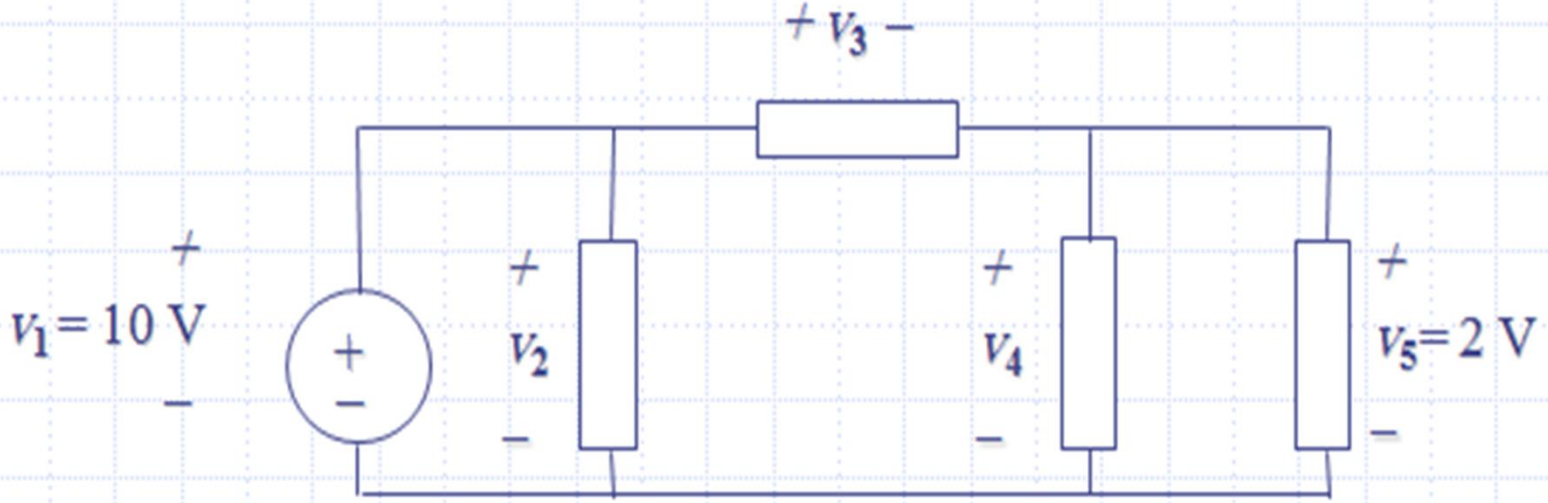
Q: If $v_1 = 10$ V and $v_5 = 2$ V, what are v_2, v_3, and v_4?

A:

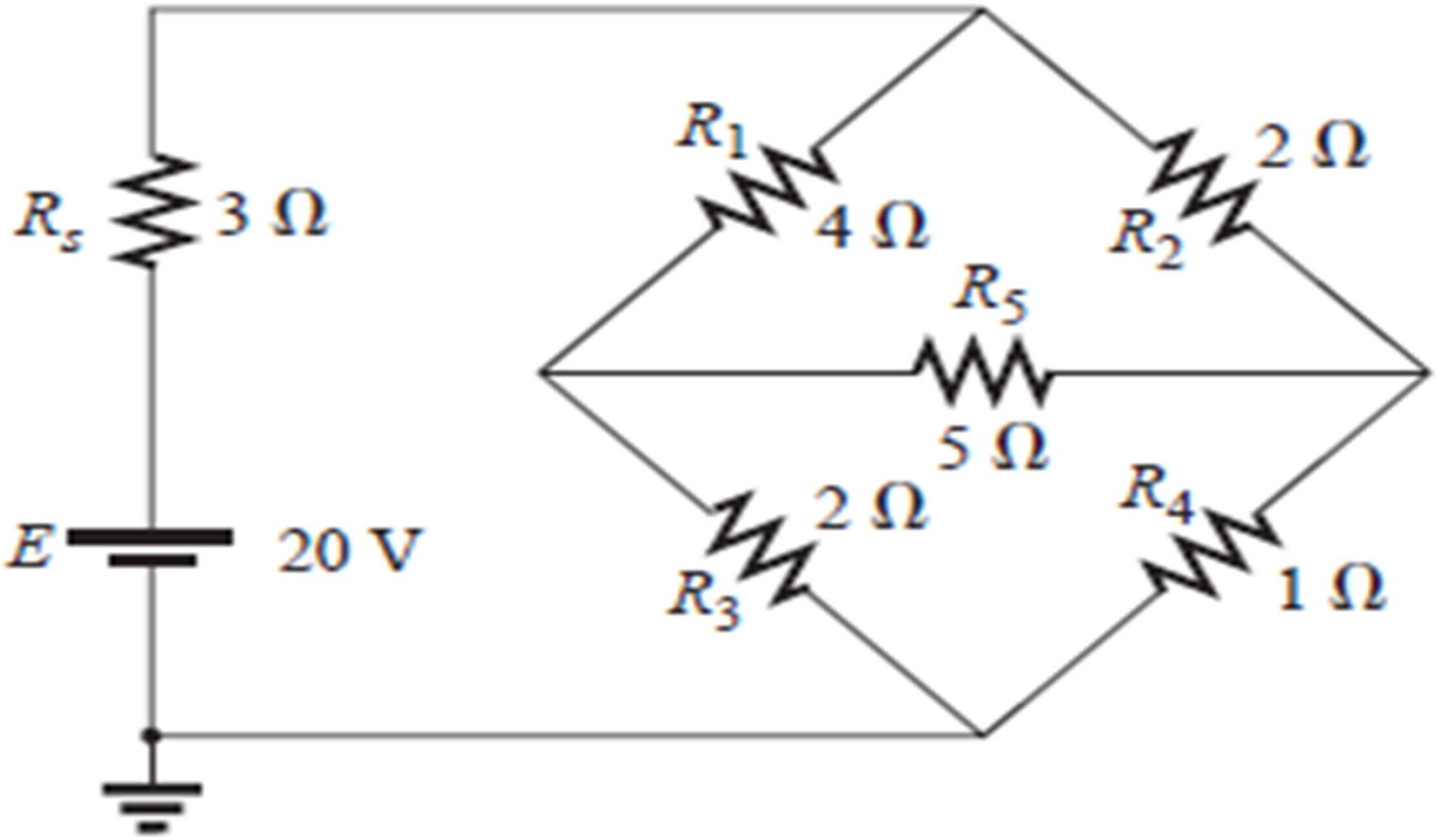
$v_2 = 10$ V

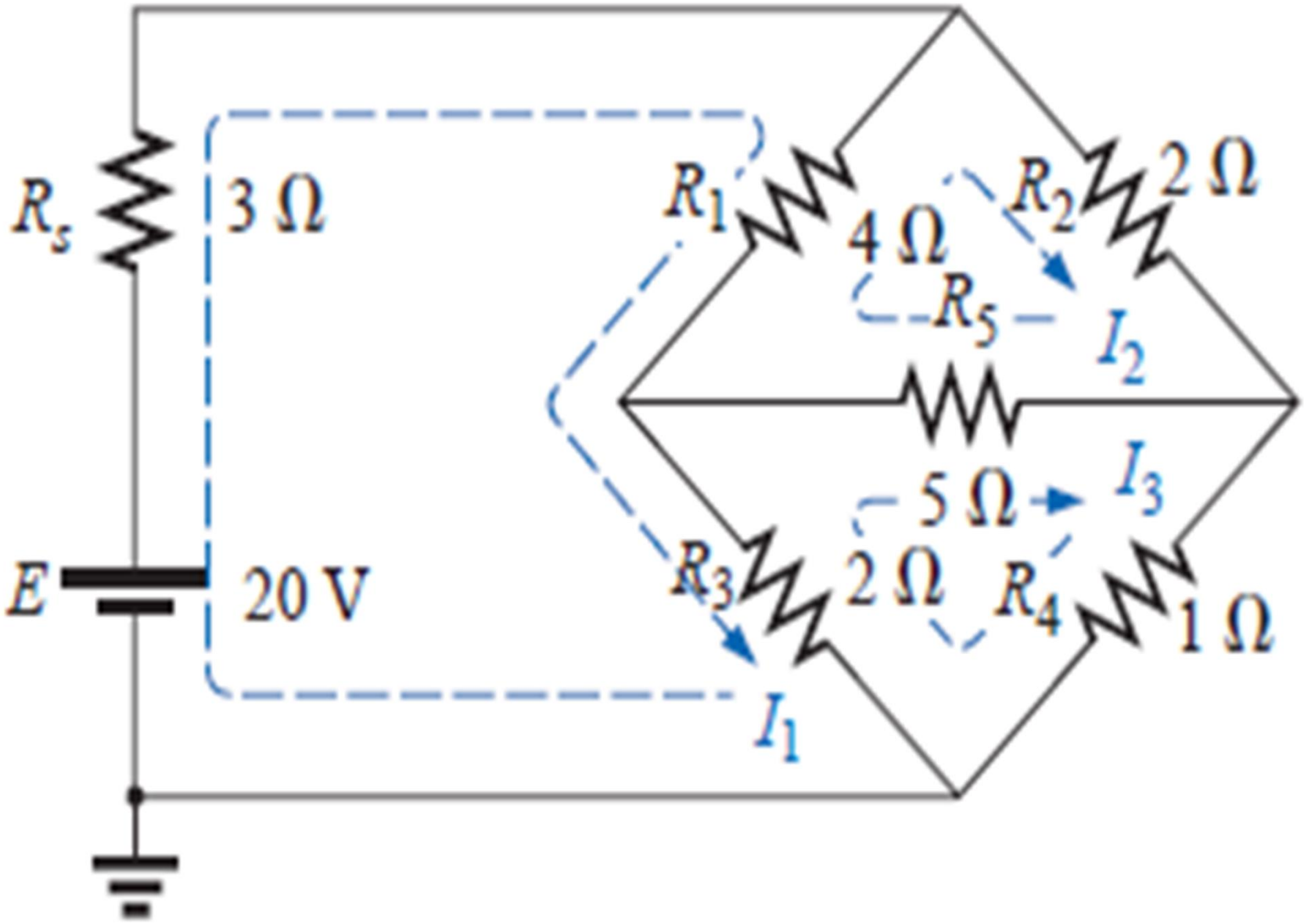
$v_3 = 10\text{ V} - 2\text{ V} = 8$ V

$v_4 = 2$ V



Example: 3 find loop currents




R_s
3 Ω
E
20 V
R_1
4 Ω
R_2
2 Ω
R_5
5 Ω
R_3
2 Ω
R_4
1 Ω
I_1
I_2
I_3

$$(3\ \Omega + 4\ \Omega + 2\ \Omega)I_1 - (4\ \Omega)I_2 - (2\ \Omega)I_3 = 20\ \text{V}$$
$$(4\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (4\ \Omega)I_1 - (5\ \Omega)I_3 = 0$$
$$(2\ \Omega + 5\ \Omega + 1\ \Omega)I_3 - (2\ \Omega)I_1 - (5\ \Omega)I_2 = 0$$

and

$$\begin{aligned} 9I_1 - 4I_2 - 2I_3 &= 20 \\ -4I_1 + 11I_2 - 5I_3 &= 0 \\ -2I_1 - 5I_2 + 8I_3 &= 0 \end{aligned}$$

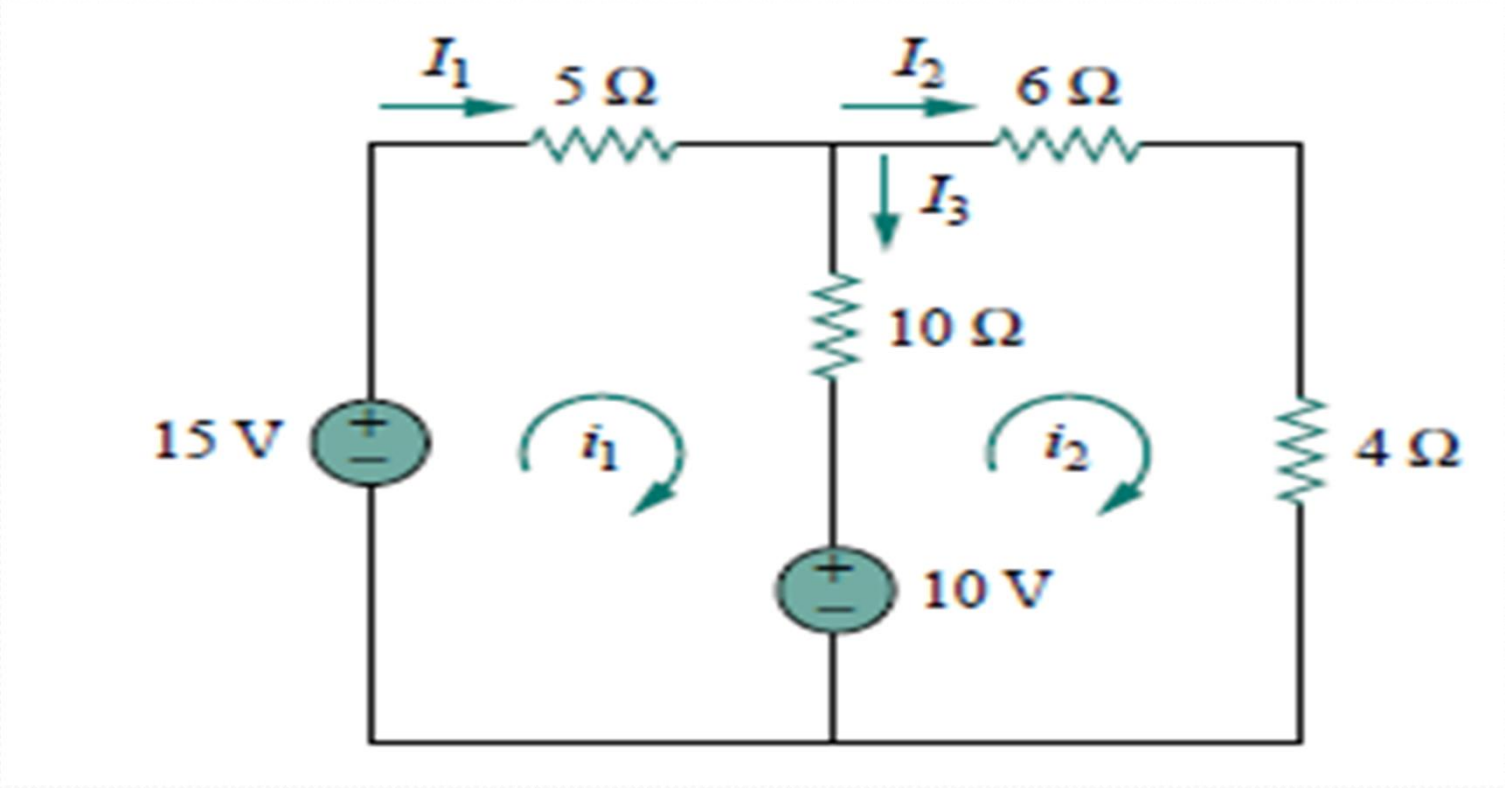
with the result that

$$I_1 = \mathbf{4\ A}$$
$$I_2 = \mathbf{2.667\ A}$$
$$I_3 = \mathbf{2.667\ A}$$

The net current through the 5-Ω resistor is

$$I_{5\Omega} = I_2 - I_3 = \mathbf{2.667\ A - 2.667\ A = 0\ A}$$

Example: 4 find branch currents



Solution:

We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \tag{3.5.1}$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \tag{3.5.2}$$

METHOD 1 Using the substitution method, we substitute Eq. (3.5.2) into Eq. (3.5.1), and write

$$6i_2 - 3 - 2i_2 = 1 \quad \Longrightarrow \quad i_2 = 1 \text{ A}$$

From Eq. (3.5.2), $i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

METHOD 2 To use Cramer's rule, we cast Eqs. (3.5.1) and (3.5.2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

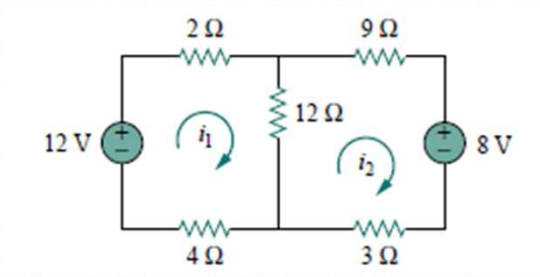
Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

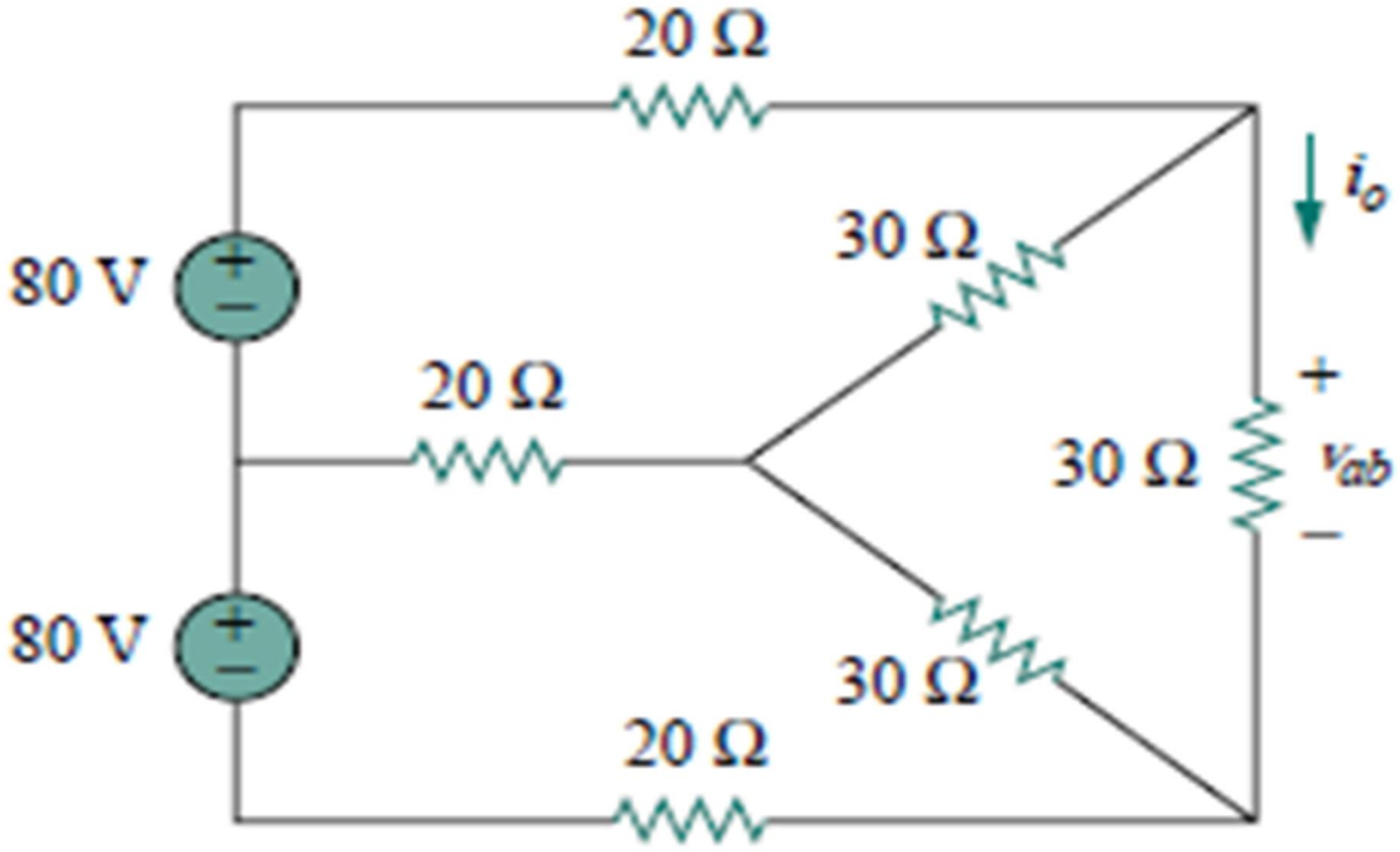
as before.

Example: 5 - find loop currents

H.W -1



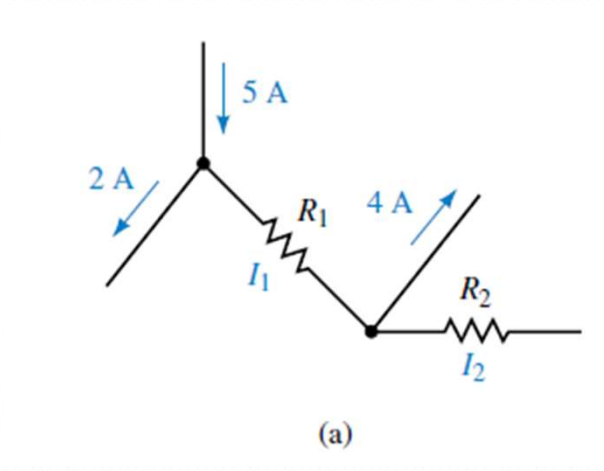
Example: 6 - find io , (H.W -2) Vab



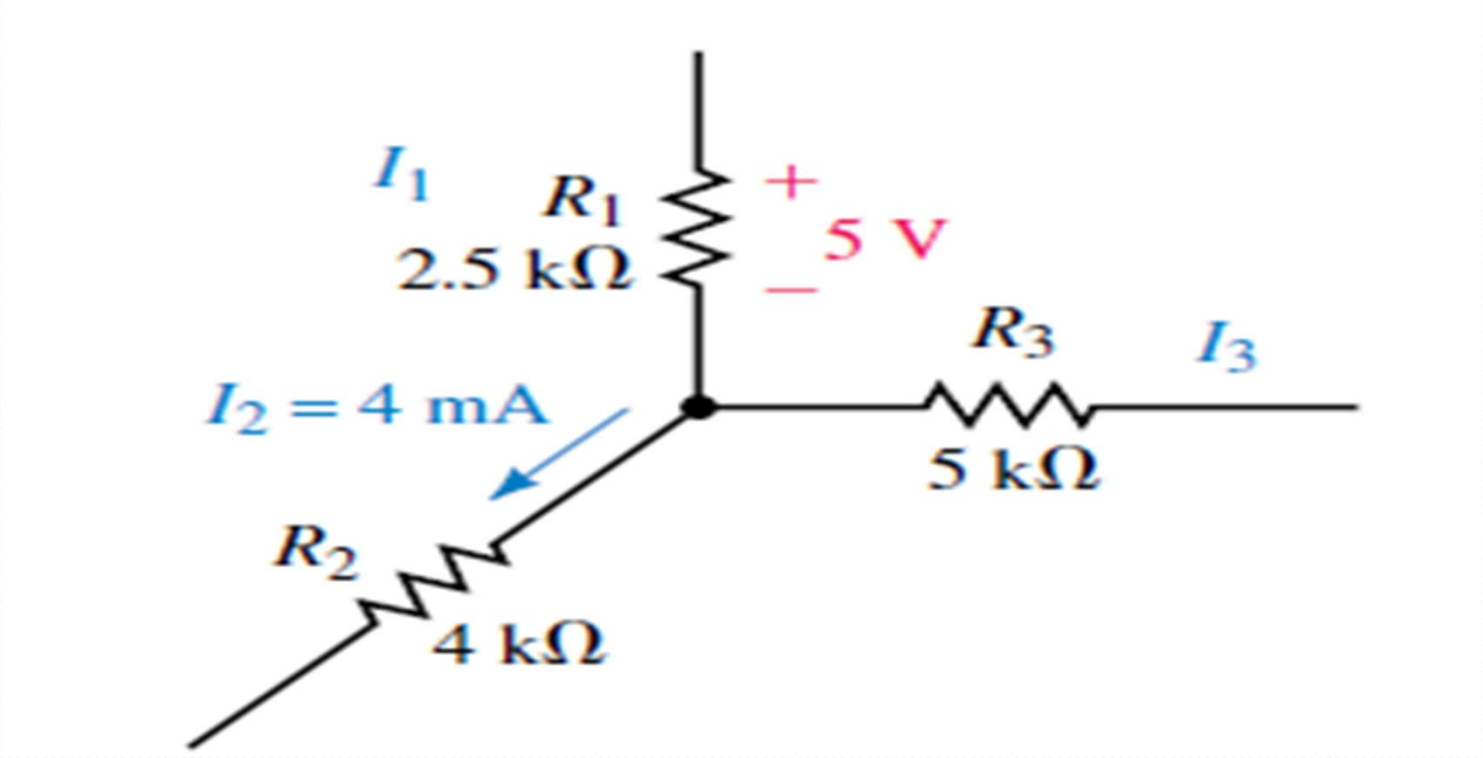
Example : 7 - find the magnitude and the directions of the unknown current

$$I_1 = 5 - 2 = 3\ A$$

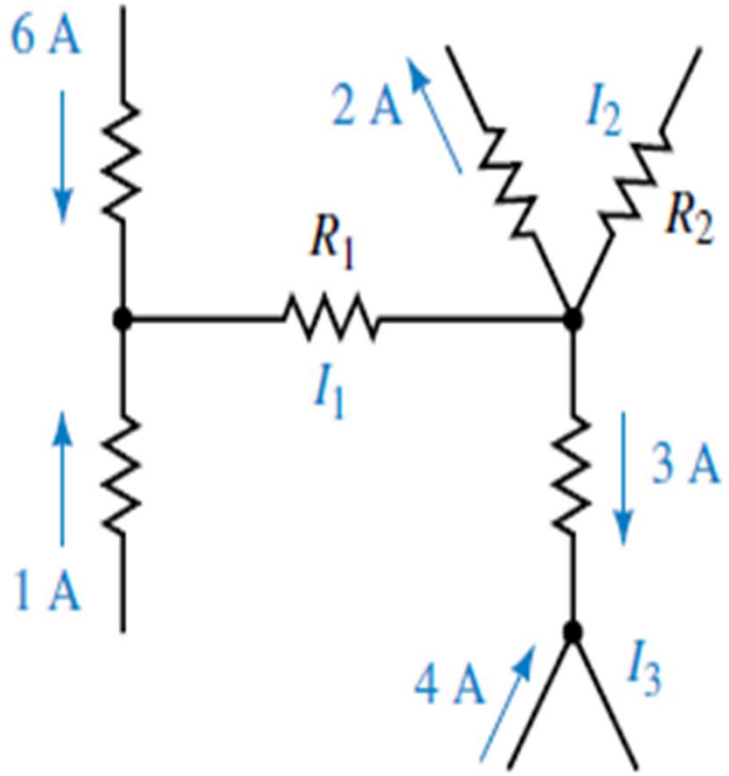
$$I_2 = 4 - I_1 = 4-3 = 1\ A$$



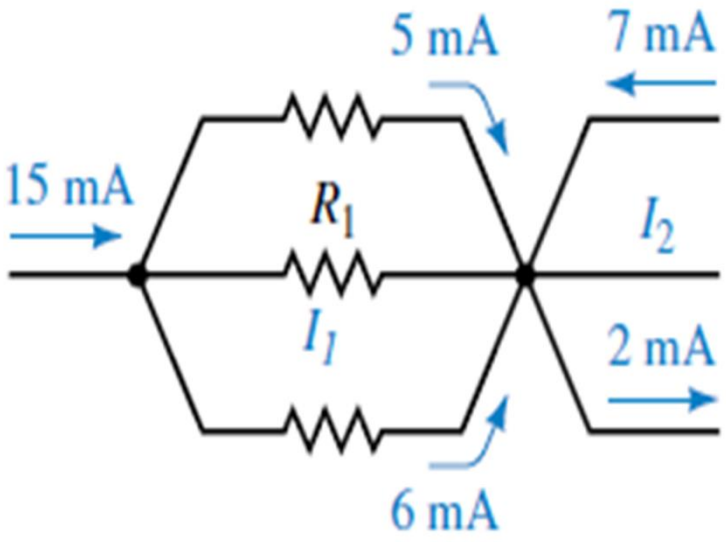
Example : 8 - find the magnitude and the directions of the unknown current (H.W-3)–



Example : 9 - find the magnitude and the directions of the unknown currents(H.W-4)



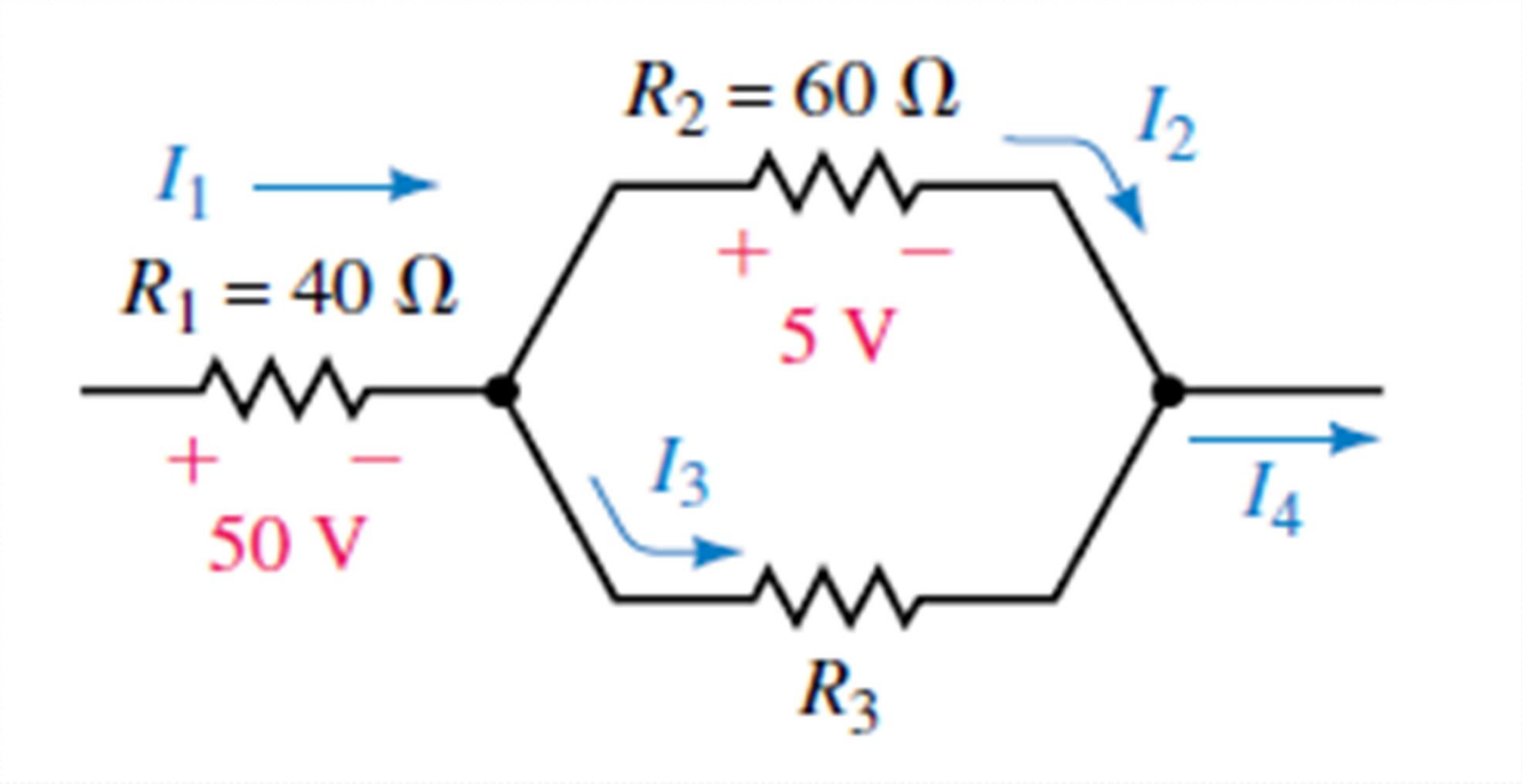
(b)



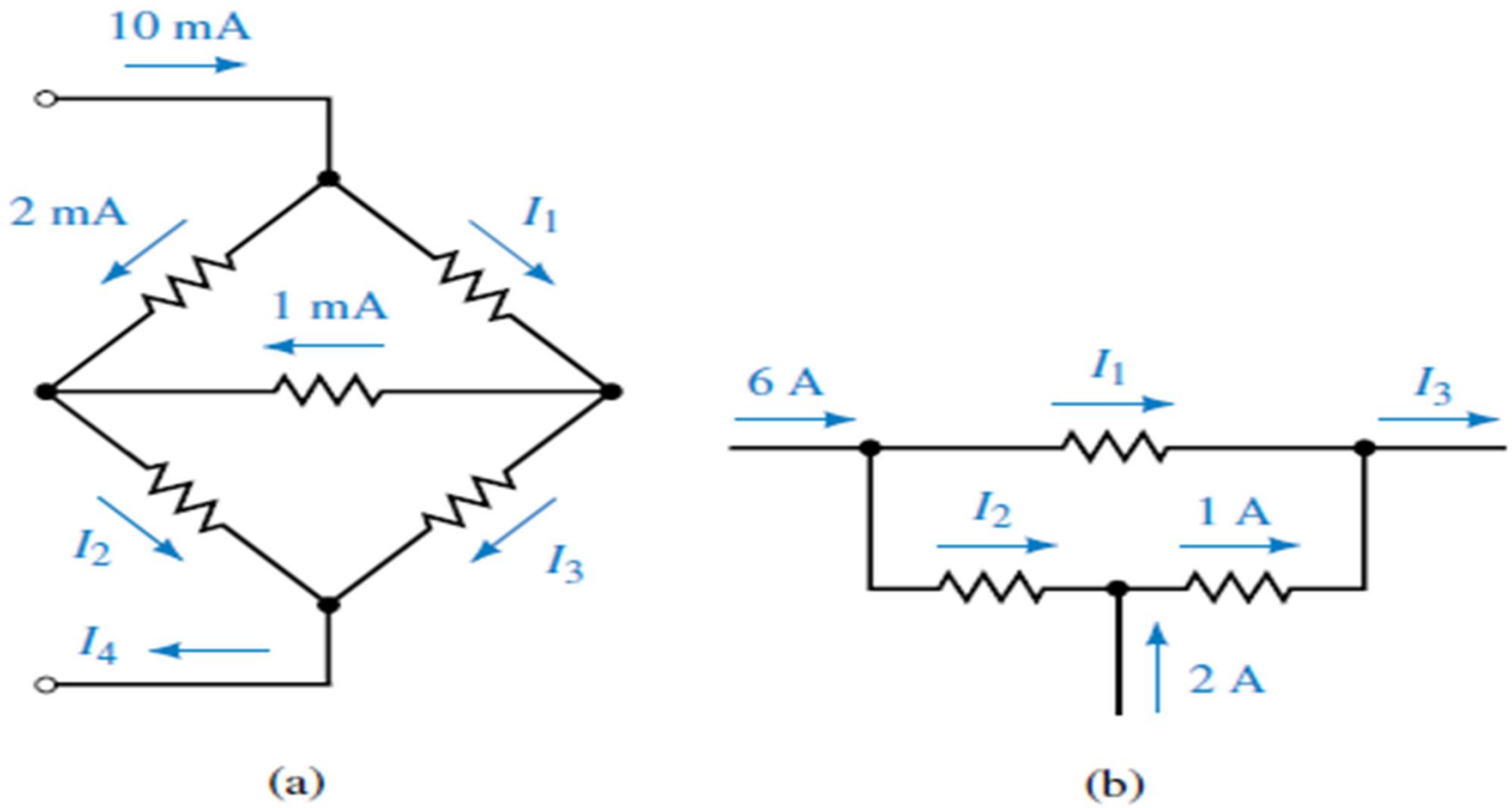
(c)

Example : 10- find the magnitude and the directions of the unknown current and voltage

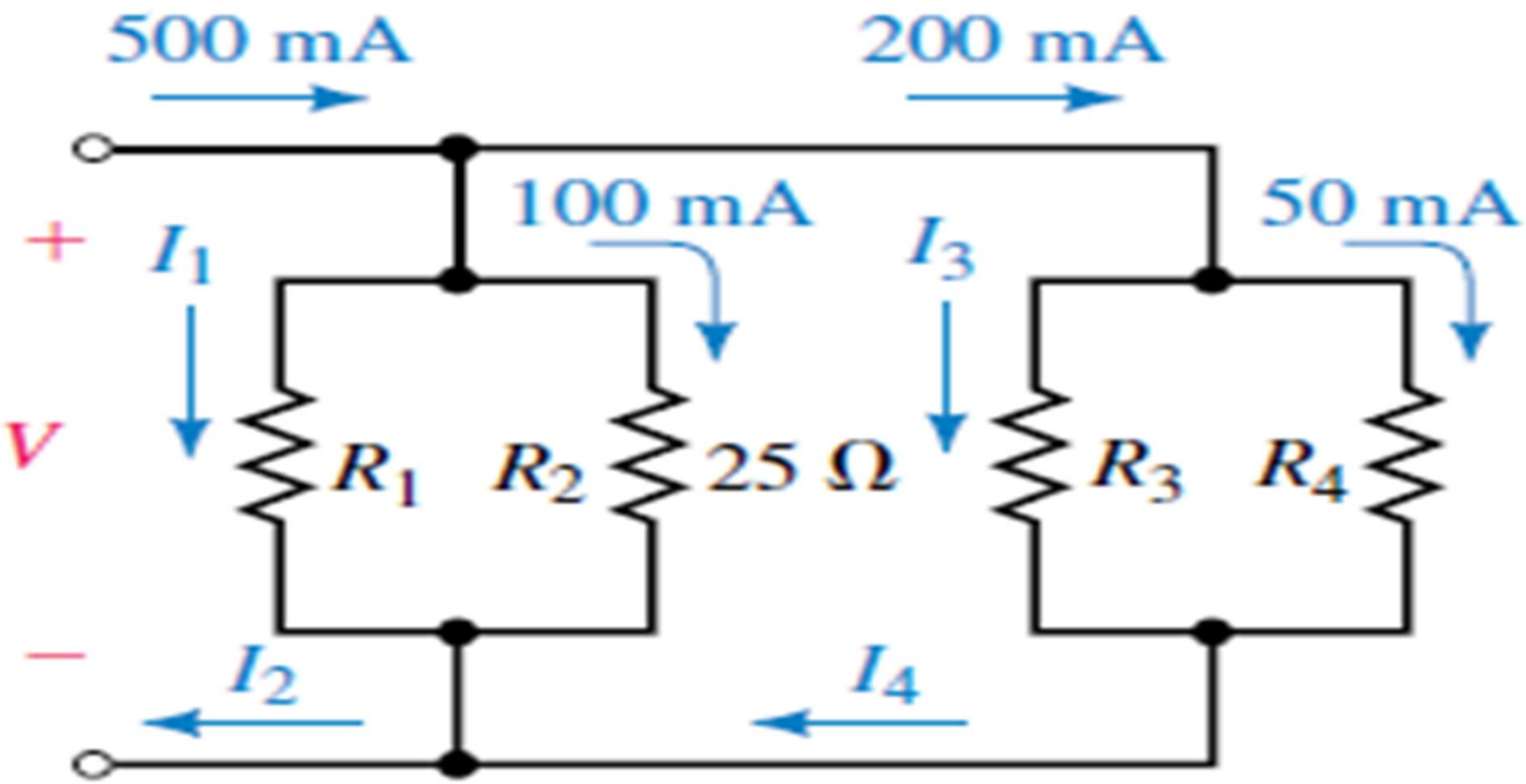
H.W-5



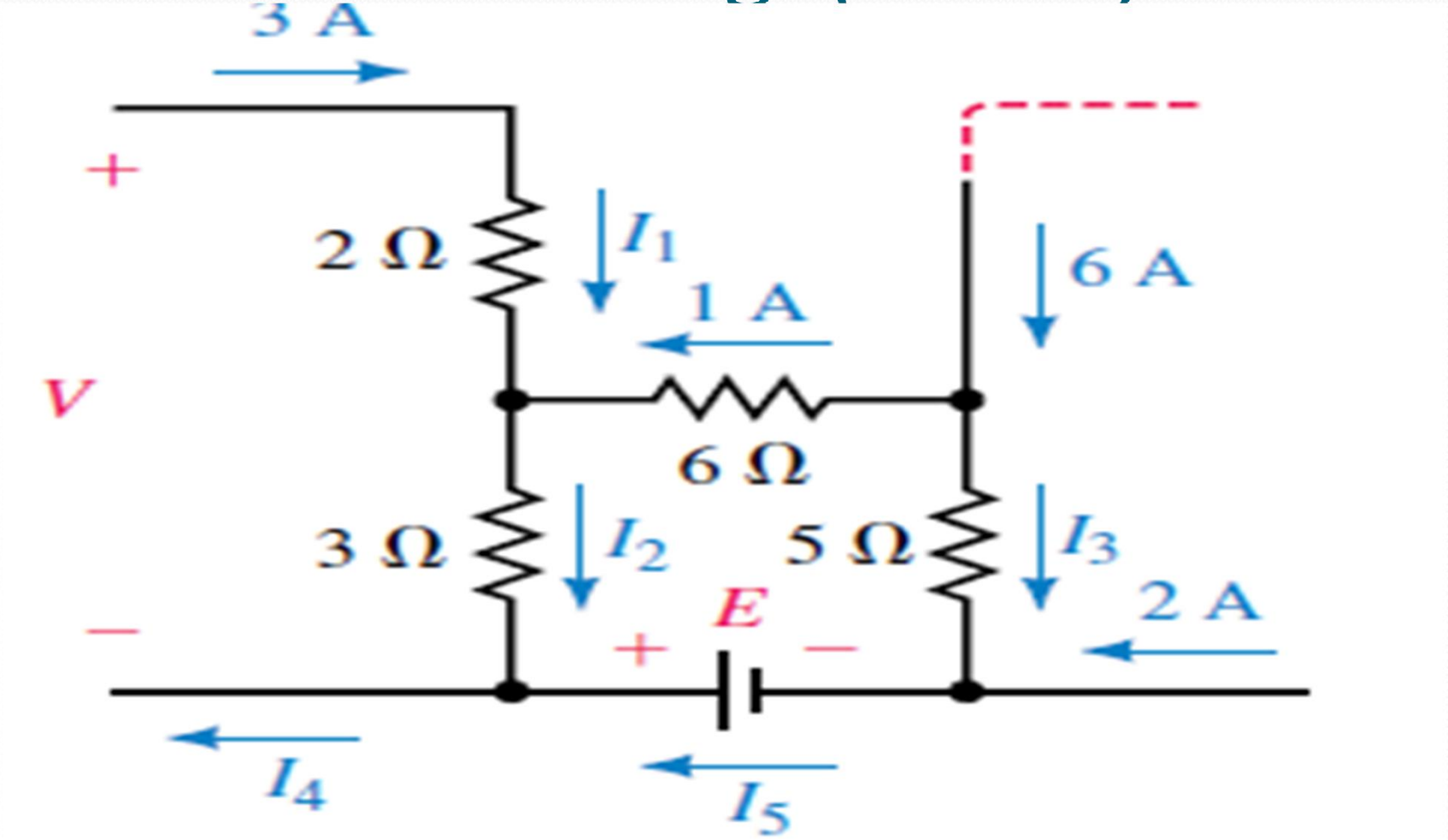
Example 1.12 Find the magnitude and the directions of the unknown current(H.W-6)



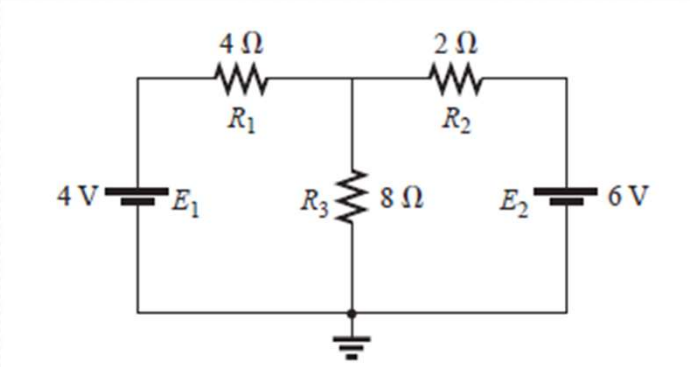
Example : 12 - find the magnitude and the directions of the unknown current and voltage(h.w -7)



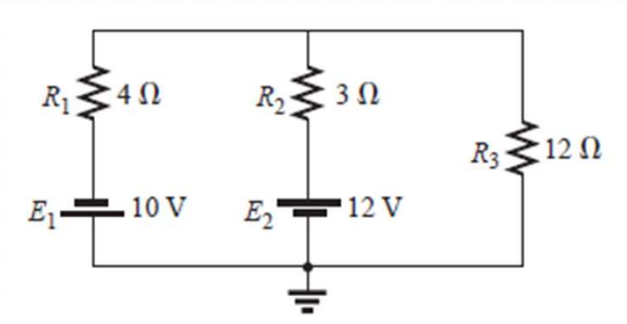
Example : 13 - find the magnitude and the directions of the unknown current and voltage(H.W-8)



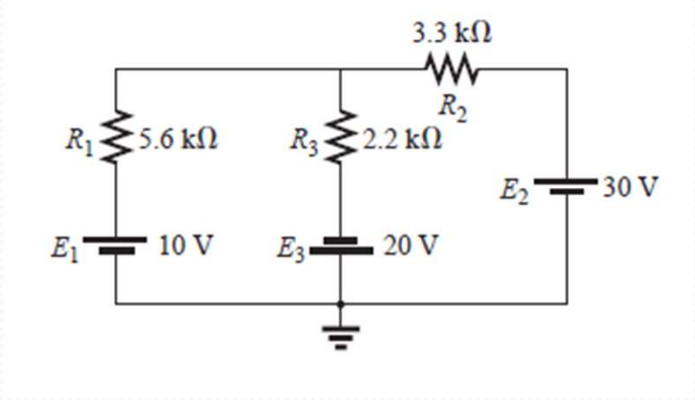
Example : 14 - for the circuit H. 5 find the branch currents()shown



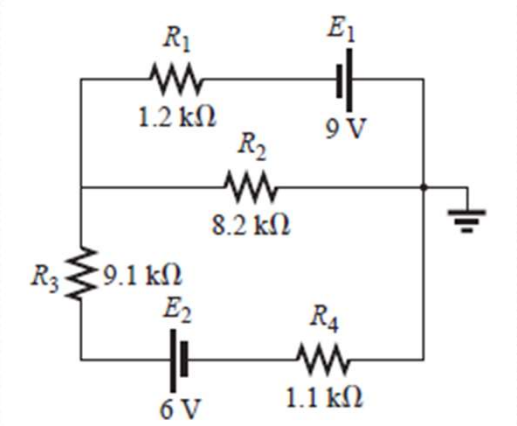
Example :15 - for the circuit shown find the branch currents(h.w-10)



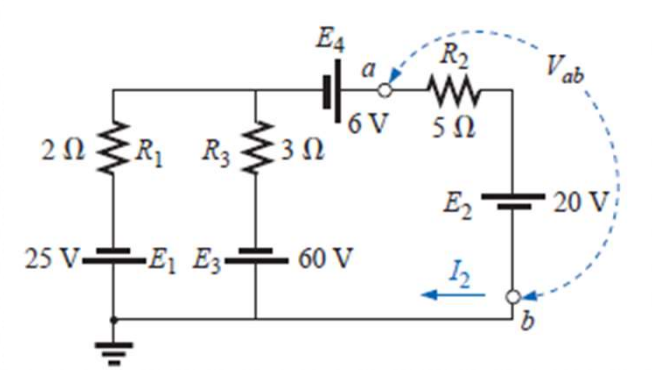
Example : 16 - for the circuit shown find the branch currents



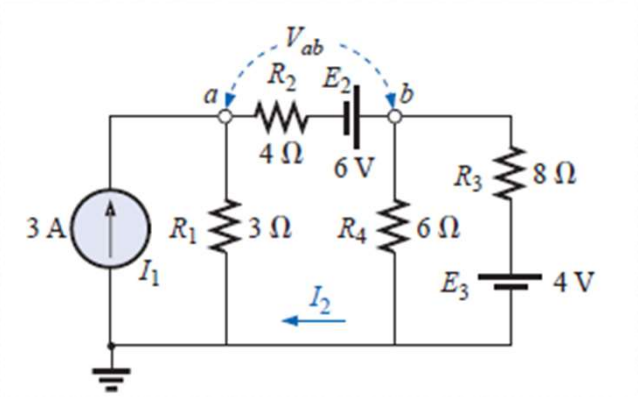
Example :17 - for the circuit shown find the branch currents



Example : 18 - for the circuit shown find the branch currents and Vab



Example : 19 - for the circuit the branch currents shown find and Vab



Example : 20 - for the circuit shown find the branch currents and Va