ألأفكار ألمركزية

1 – Thevenin,s Theorem

2 – Norton , Theorem

3 - Source Transformation Theorem

4 – Superposition Theorem

5 – Max. power Transfer Theorem

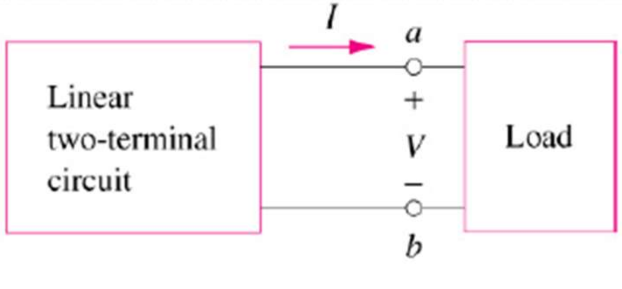
6 - Example

Thevenin,s theorem

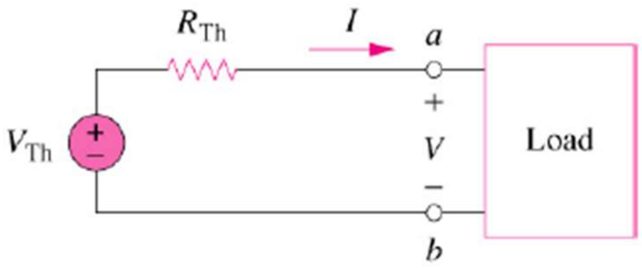
It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a voltage source V_{TH} in series with a resistor R_{TH},

where

- V_{Th} is the open-circuit voltage at the terminals.
- R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



(a)



(b)

How to Find thevenin,s equivalent circuit

A - How to find (Rth)

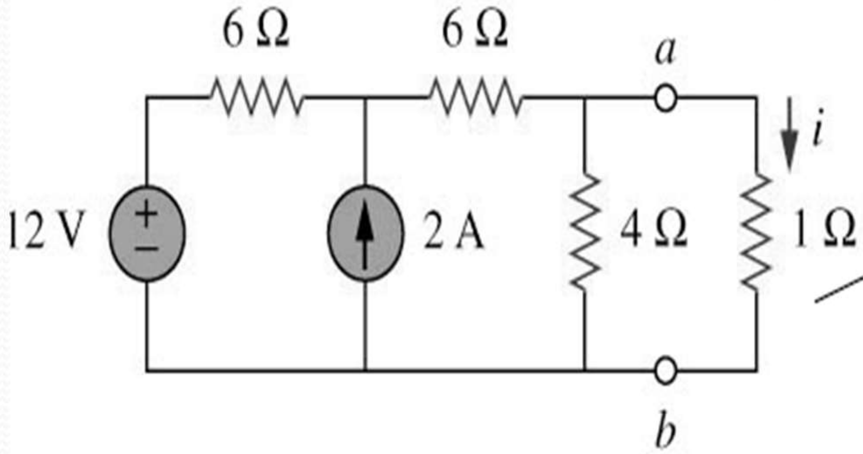
- 1- Remove that portion of the network
- 2-Mark the terminal of the remaining network
- 3-Set all sources to zero(voltage source short circuit and current source open cicuit)
- 4-calculate (Rth) resistance

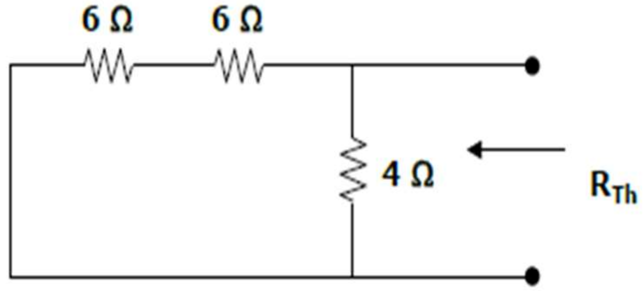
B- How to find (Vth)

- 1- Returning all sources to their origin position
- 2- Remove that portion of the Network
- 3- Calculate the open circuit voltage between the marked terminal

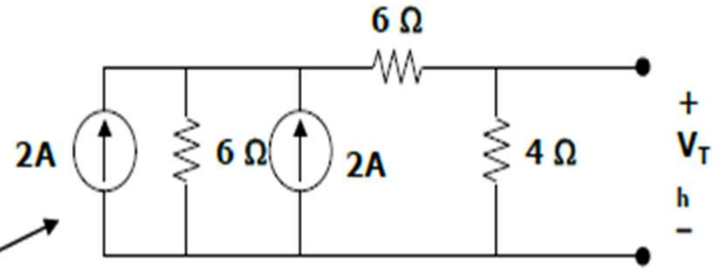
Example

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below. Hence find i.





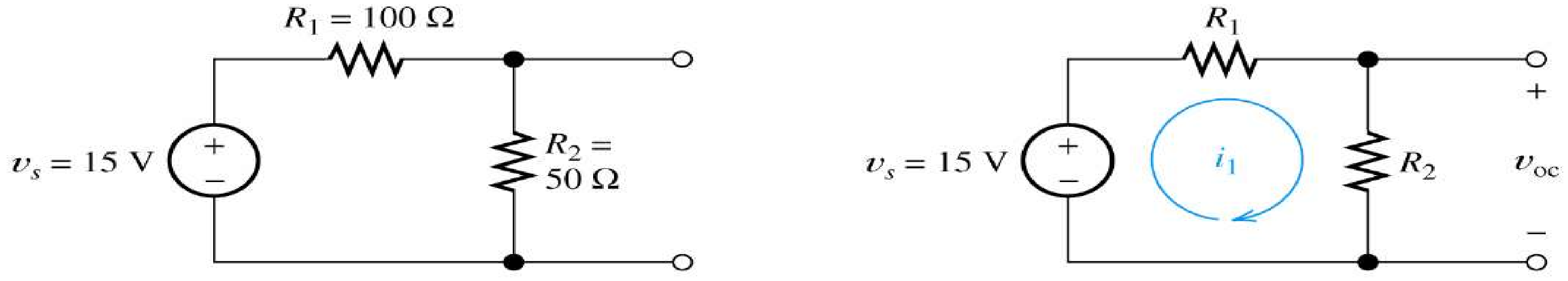
(a)



(b)

Answer: $V_{TH} = 6V$, $R_{TH} = 3\Omega$, $i = 1.5A$

EX – 1 for the circuit shown in fig-a Find the Thevenin Equivalent Circuit

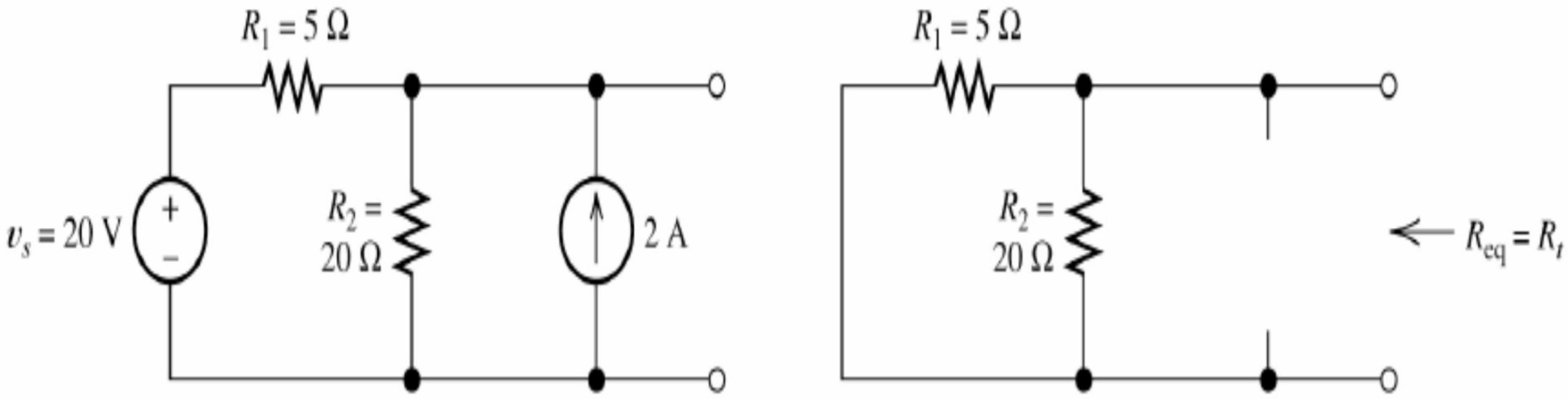


(a) Original circuit (b) Analysis with an open circuit

A- Use voltage divider formed by R_1 and R_2 to find v_{oc}: or Vth

$$v_{oc} = \frac{R_2}{R_1 + R_2} v_s = \frac{50}{100 + 50} 15V = 5V$$

A- Find Thevenin ,s Resistance

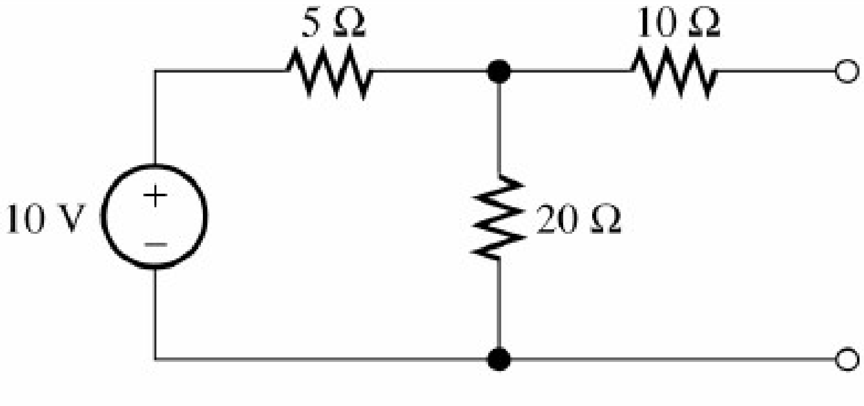


(a) Original circuit

(b) Circuit with sources zeroed

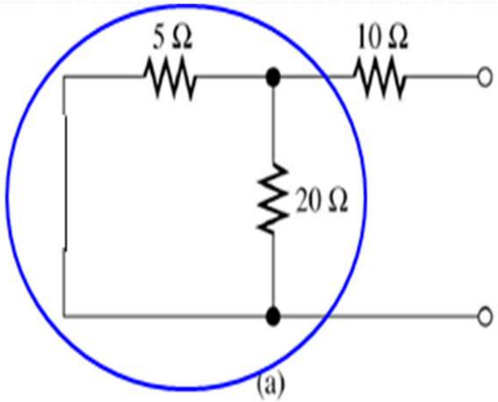
$$R_{eq} = R_t = \frac{R_1 R_2}{R_1 + R_2} = \frac{100}{25} = 4\Omega$$

A - Find the Thevenin Resistance



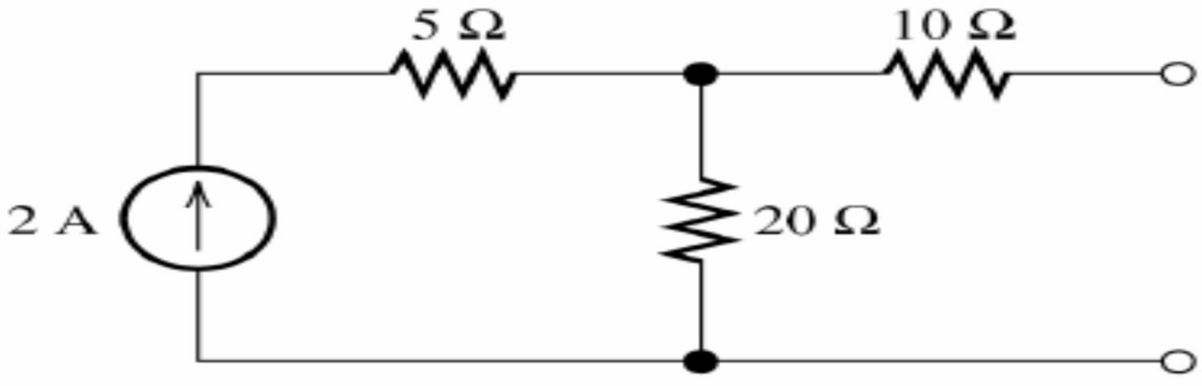
(a)

Zero the voltage source



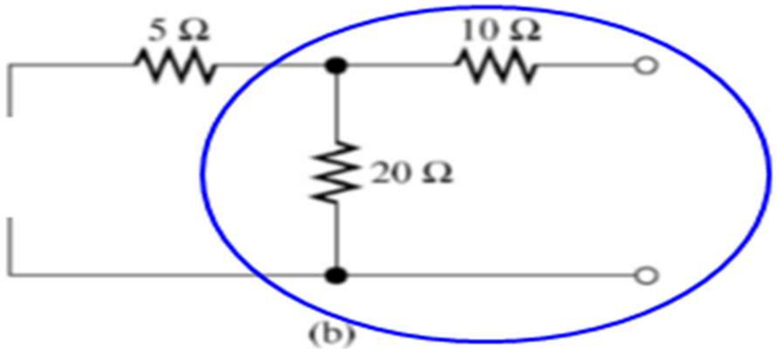
$$R_t = 10 + \frac{(5)(20)}{5 + 20} = 10 + 4 = 14\,\Omega$$

B-Find the Thevenin Resistance



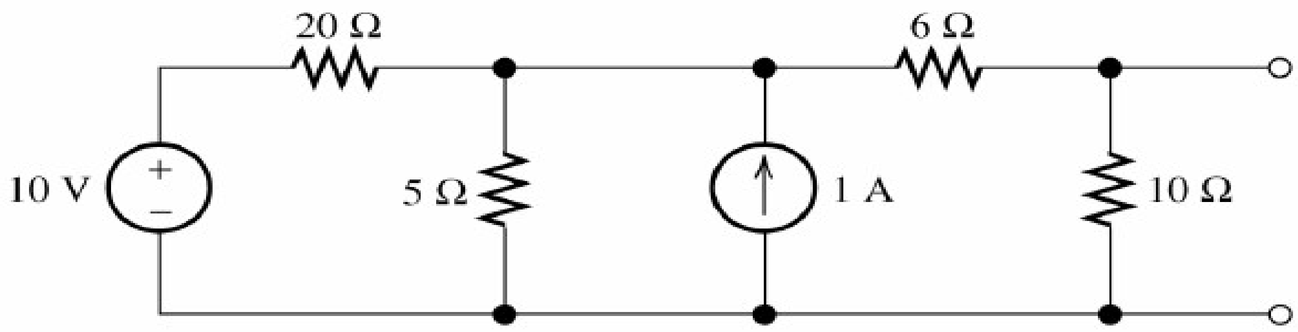
(b)

Zero the current source

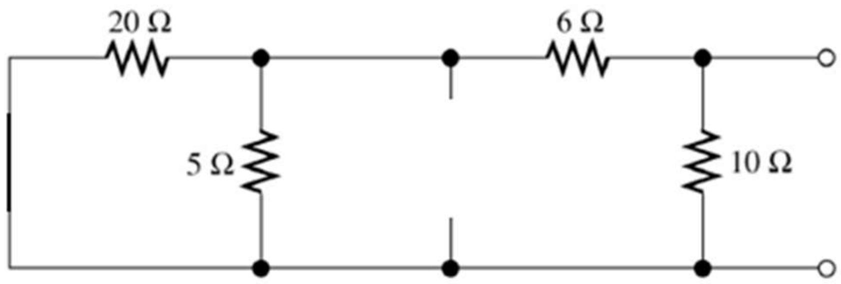


$$R_t = 10 + 20 = 30\Omega$$

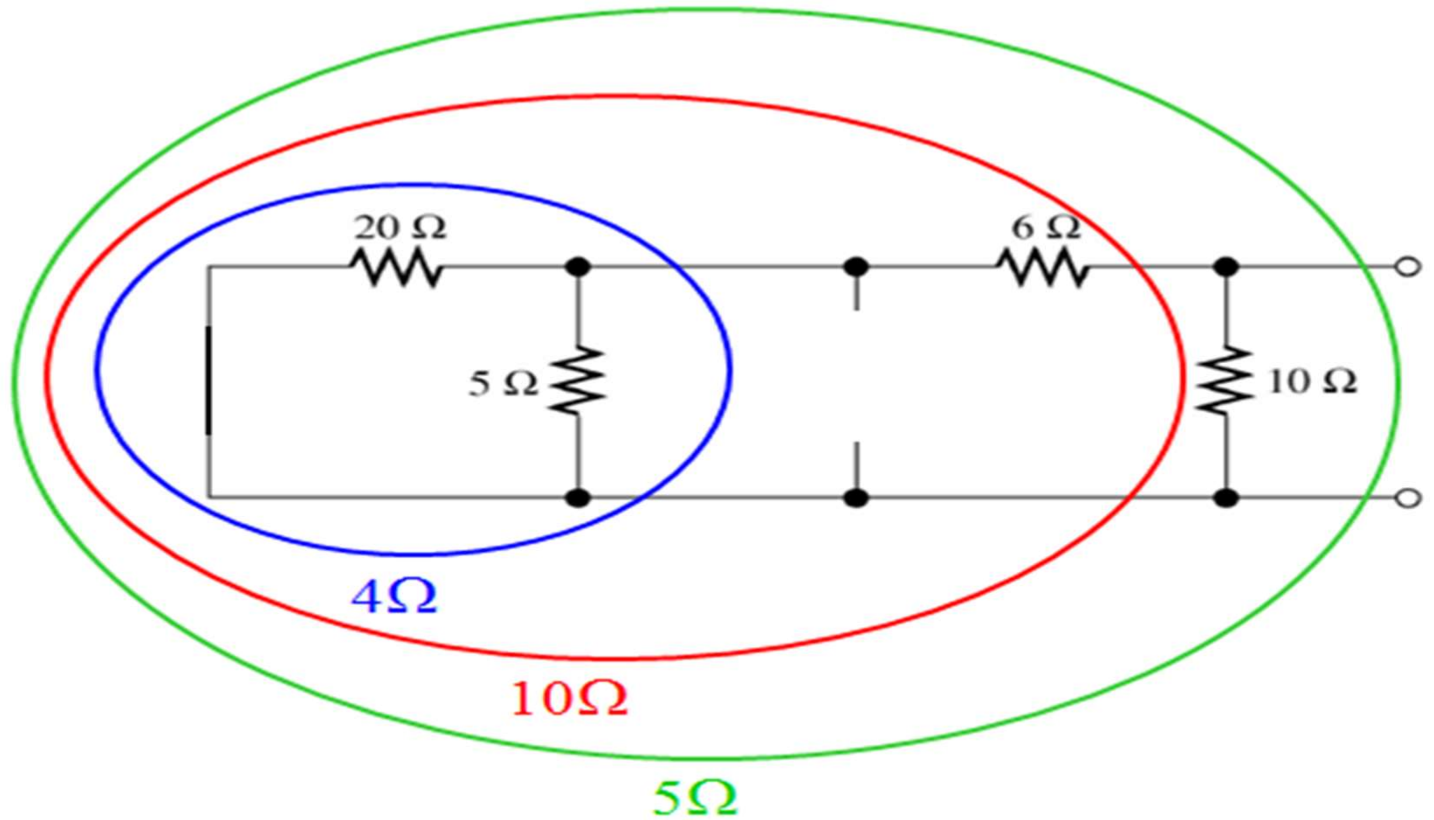
C - For the circuit shown find Rth.



Zero the current and voltage sources



Rth = 5 Ω



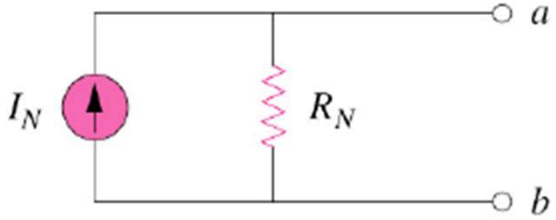
Norton,s theorem

It states that a linear two-terminal circuit can be replaced by an equivalent circuit of a current source I_N in parallel with a resistor R_N,

Where

- I_N is the short circuit current through the terminals.
- R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

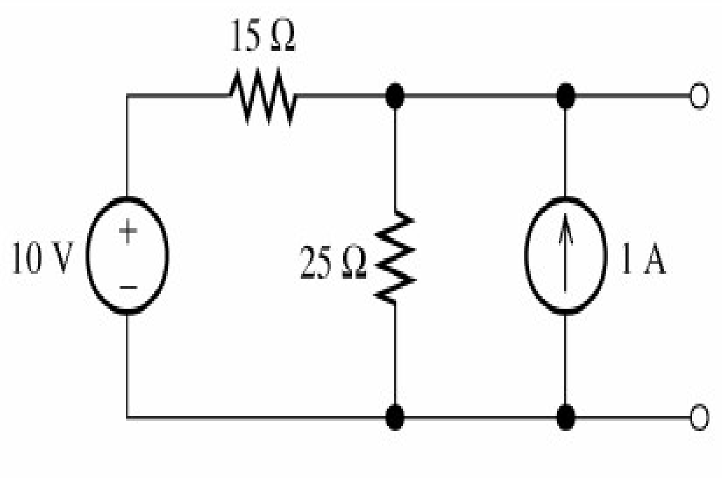
(a)



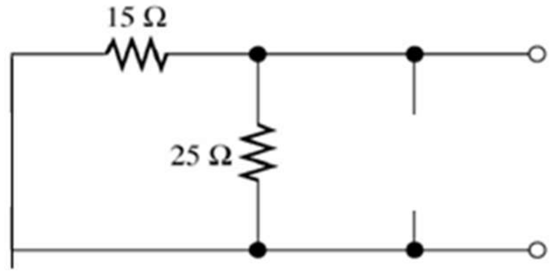
(b)

The Thevenin's and Norton equivalent circuits are related by a source transformation.

find Norton,s Resistance



(a)

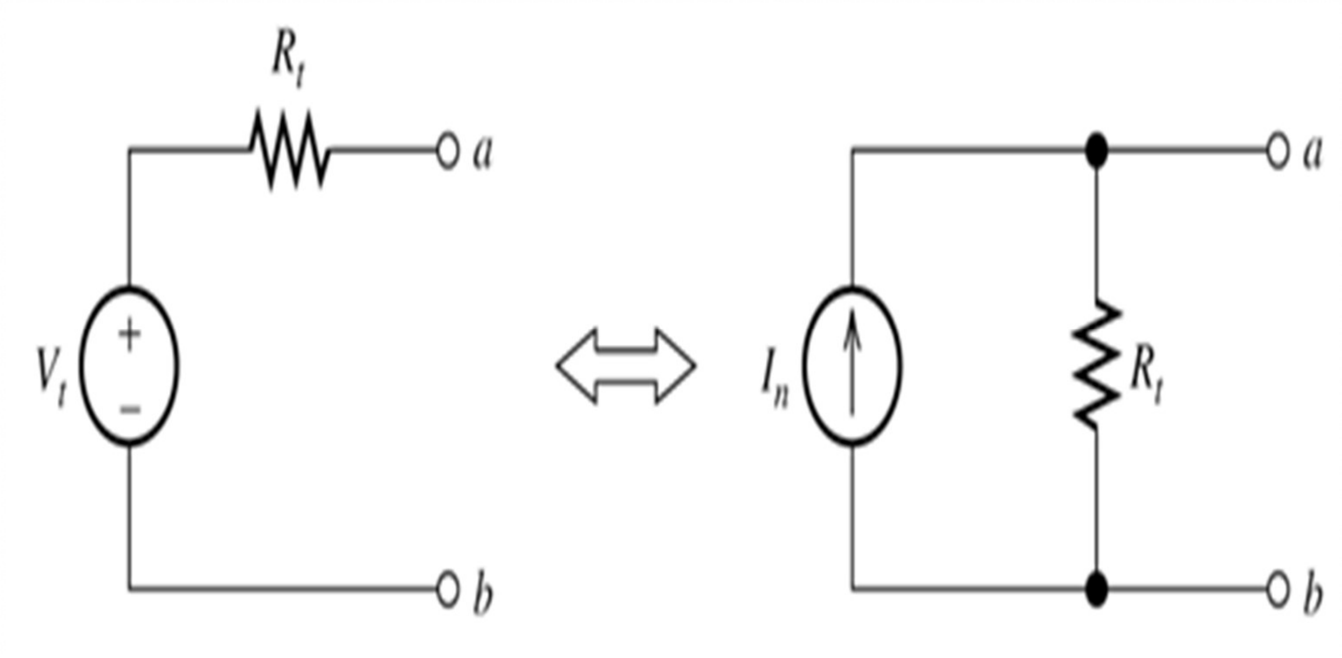


(a)

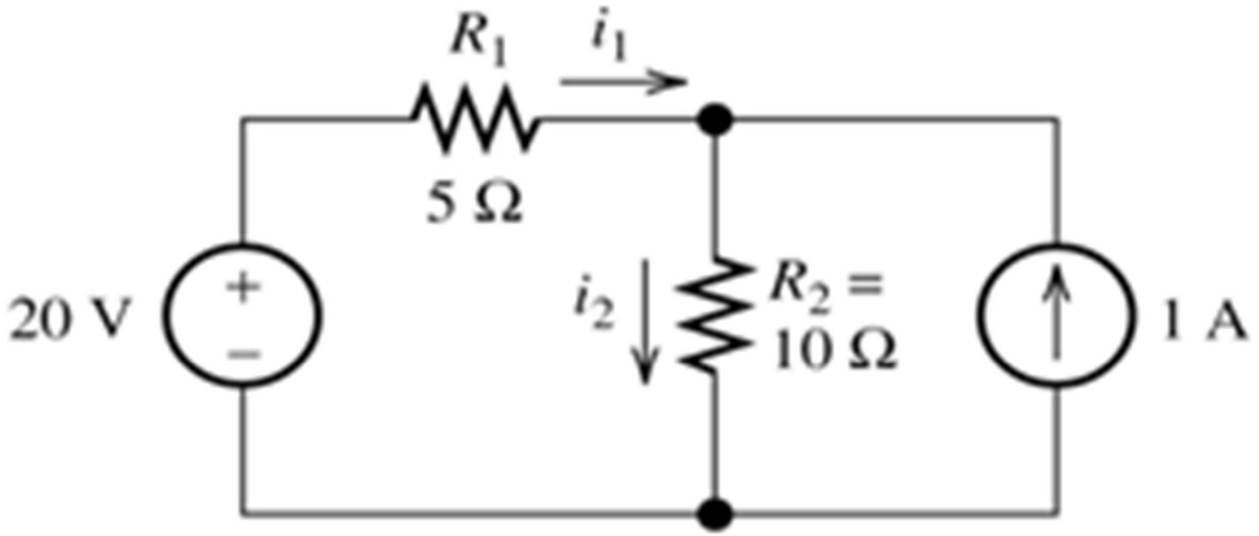
To find R_t zero the sources:

$$R_t = \frac{(15\Omega)(25\Omega)}{15\Omega + 25\Omega} = 9.38\Omega$$

Source Transformations

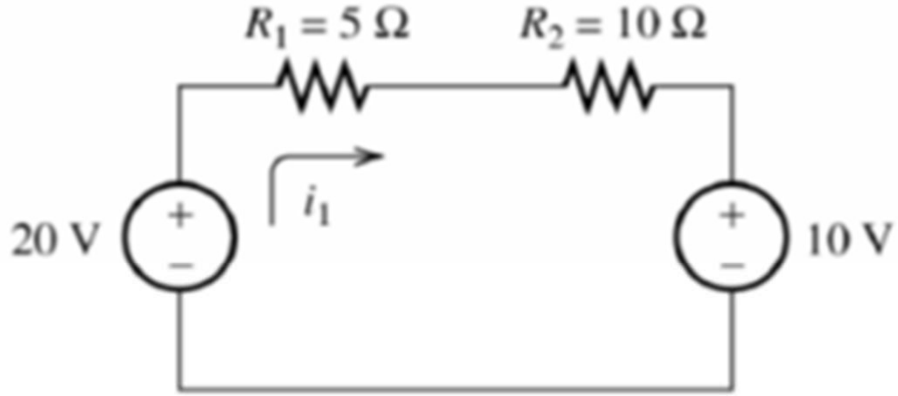


EX- 4 find the currents I1,I2 by using Source Transformations



(a) Original circuit

A- Current to Voltage Source Transformations to find I1

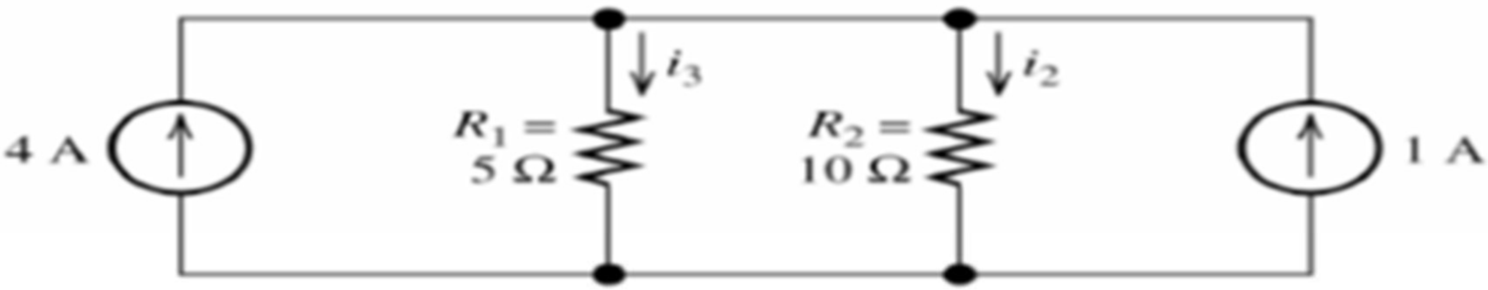


(b) Circuit after transforming the current source into a voltage source

$$V_t = i_n R_t = (1A)(10\Omega) = 10V$$

$$R_1 i_1 + R_2 i_1 + 10 - 20 = 0 \rightarrow i_1 = \frac{10}{R_1 + R_2} = 0.667A$$

B- Voltage to Current Source Transformations to find I2



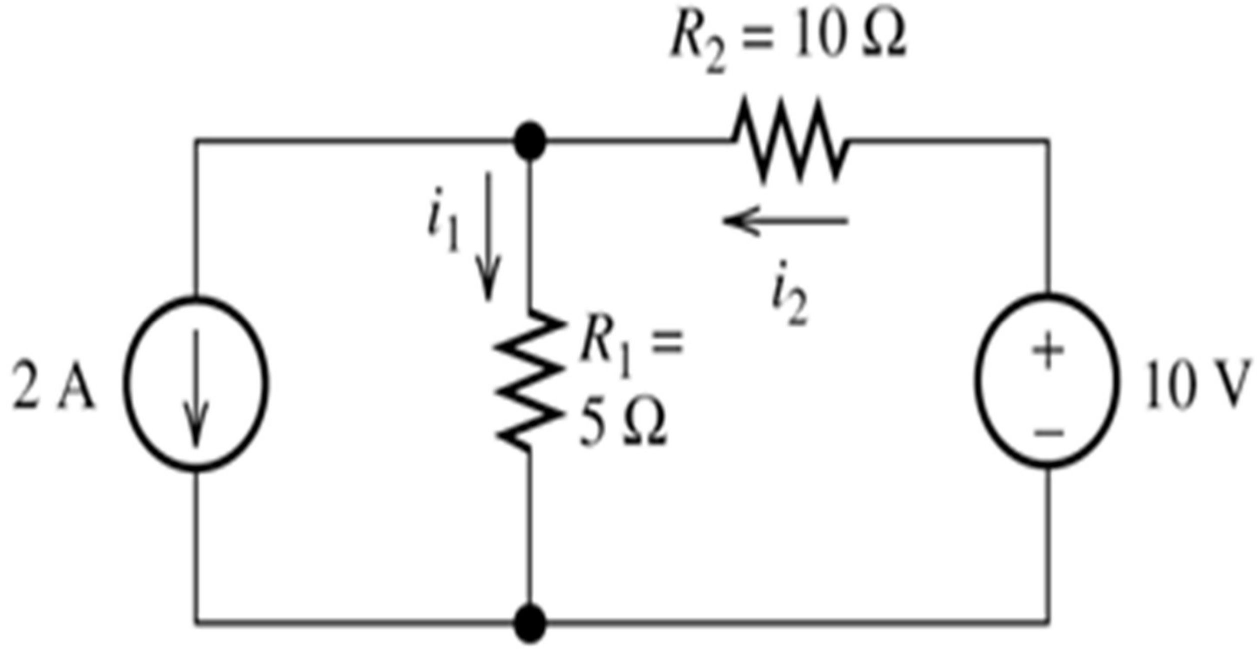
(c) Circuit after transforming the voltage source into a current source

$$i_n = \frac{V_t}{R_t} = \frac{20V}{5\Omega} = 4A$$

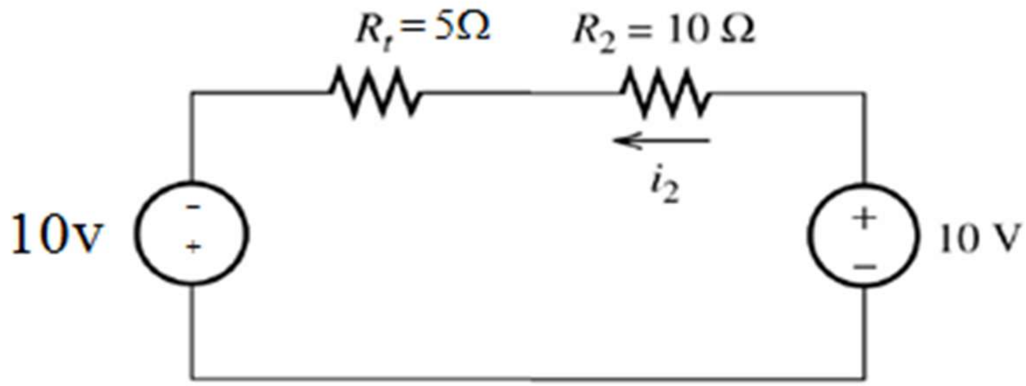
$$i_2 + i_3 = 5A$$

$$Current\ divider: i_2 = \frac{R_1}{R_1 + R_2} 5A = \frac{5}{5+10} 5A = 1.667A$$

Exam: use source transformation find the current i1 and i2



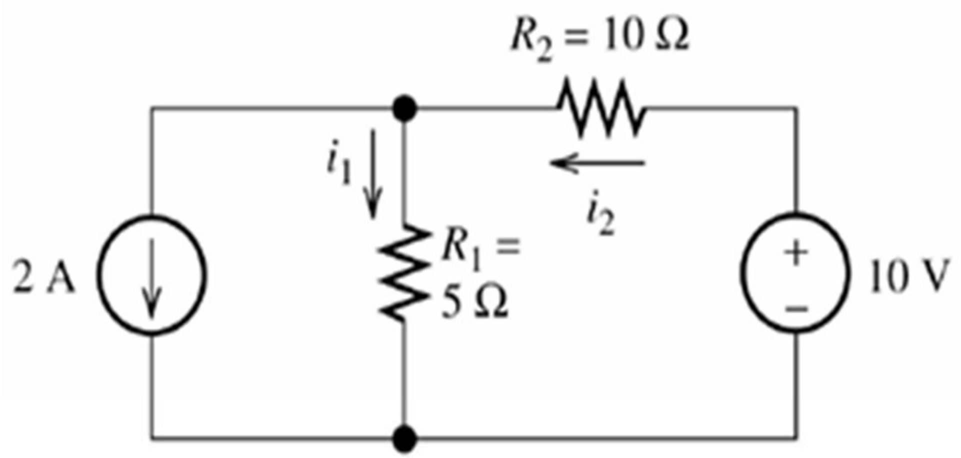
A – convert current source to voltage source to find i2 in the circuit shown



$$KVL: \quad -10V + i_2 R_2 + i_2 R_t - 10V = i_2 10 + i_2 5 - 20V = 0$$

$$i_2 = \frac{20V}{10+5} = 1.33A$$

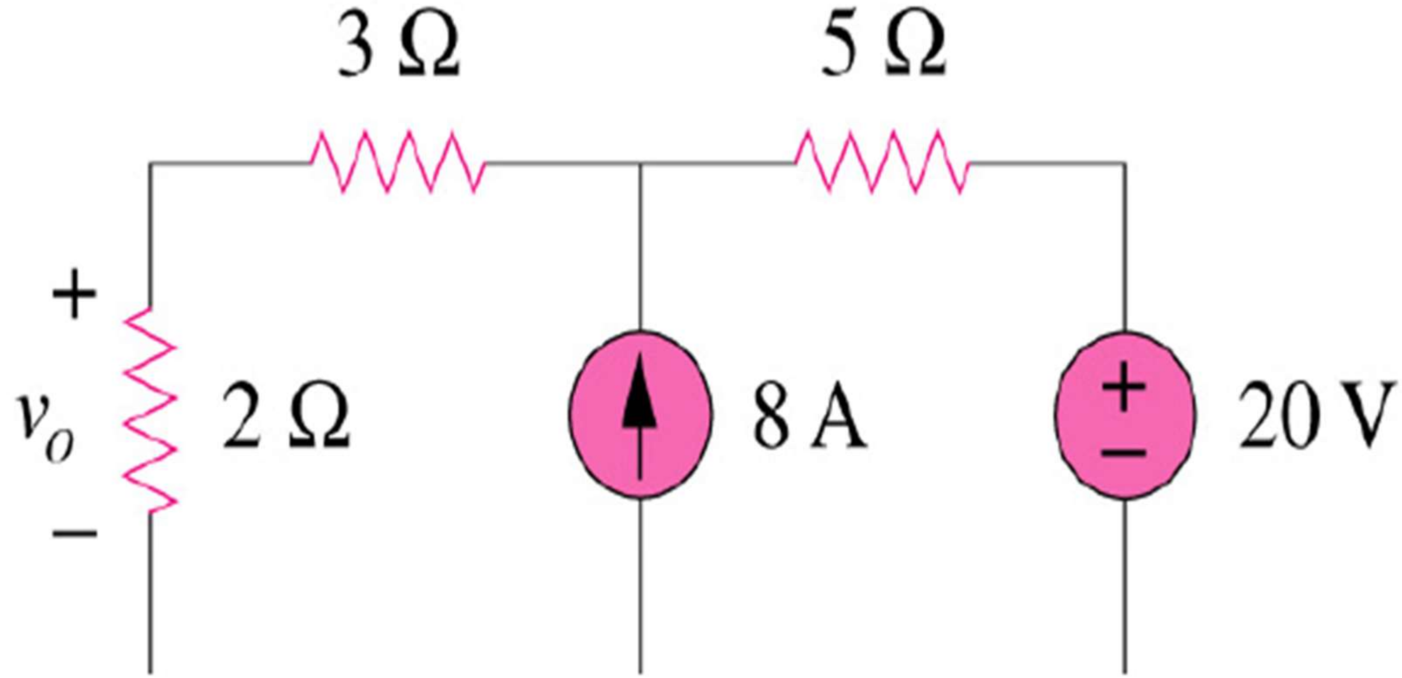
B – Find (i1) for the circuit shown



$$i_2 = i_1 + 2A \rightarrow i_1 = i_2 - 2A = 1.333 - 2A = -0.667\,A$$

Example

Find v_o in the circuit shown below using source transformation.



Superposition theorem

Steps to apply superposition principle

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. Repeat step 1 for each of the other independent sources.

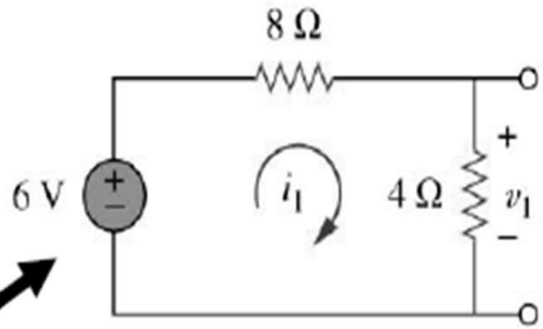
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

1. When we say turn off all other independent sources:
    - Independent voltage sources are replaced by 0 V (short circuit) and
    - Independent current sources are replaced by 0 A (open circuit).

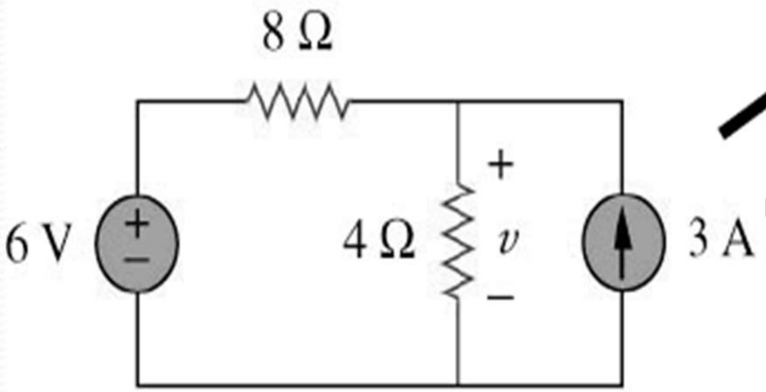
Example

Use the superposition theorem to find v in the circuit shown below.

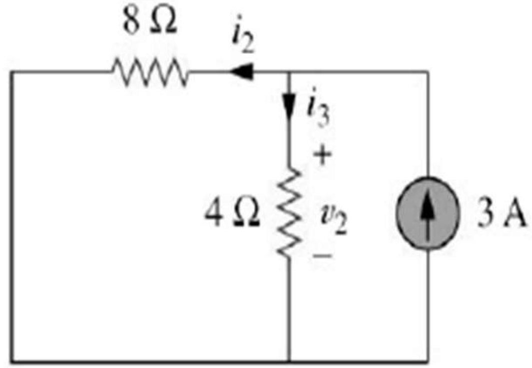
3A is discarded by open-circuit



(a)



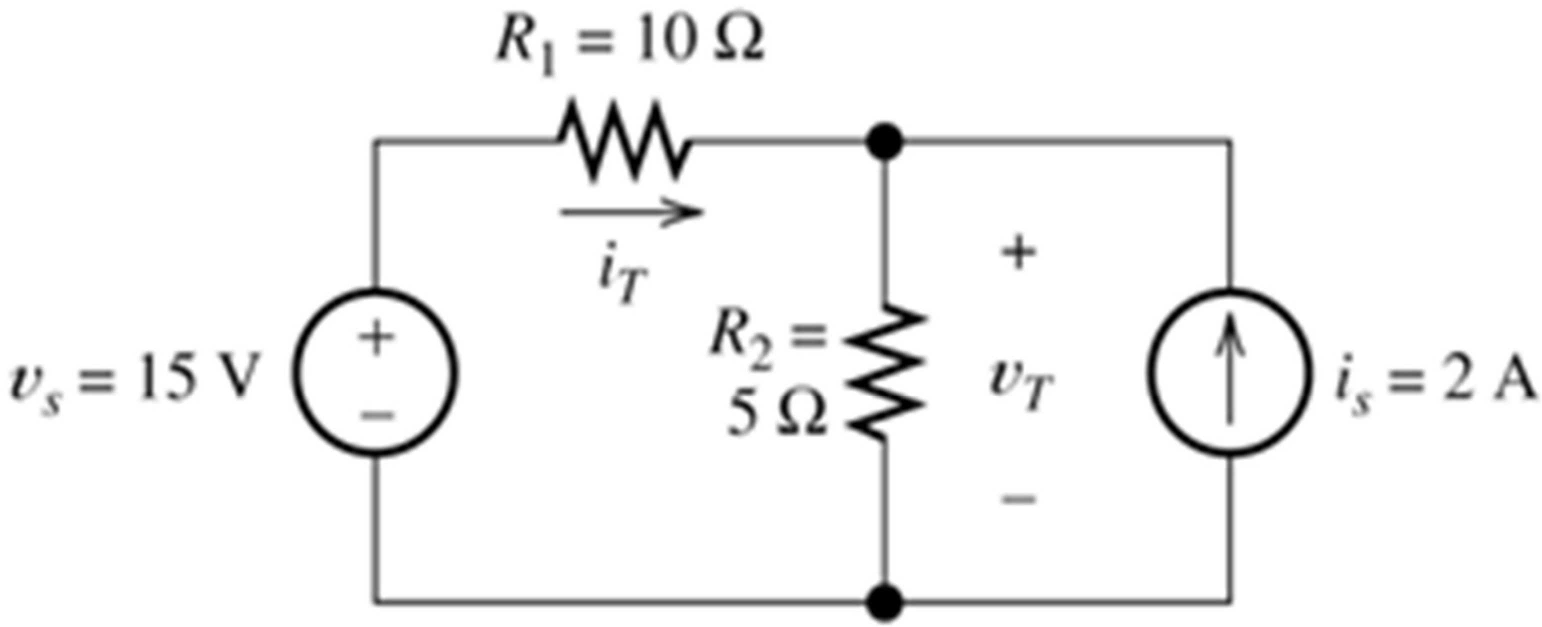
6V is discarded by short-circuit



(b)

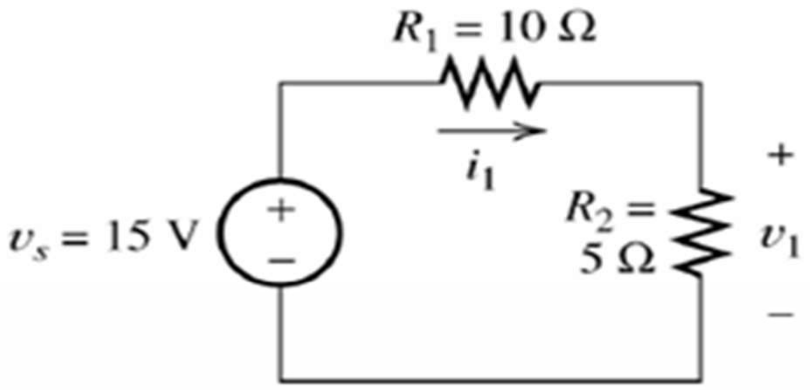
Answer: $v = 10V$

Exa : use superposition find Vt , it



(a) Original circuit

A- circuit with voltage source only



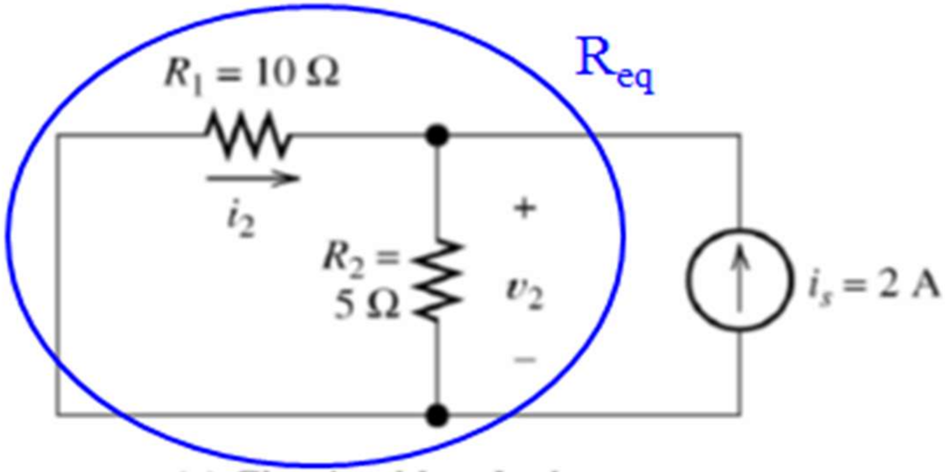
(b) Circuit with only the voltage source active

Current source open circuit

$$v_1 = \frac{R_2}{R_1 + R_2} v_s = \frac{5}{5+10} 15V = 5V$$

B- circuit with current source only

Voltage source
short circuit

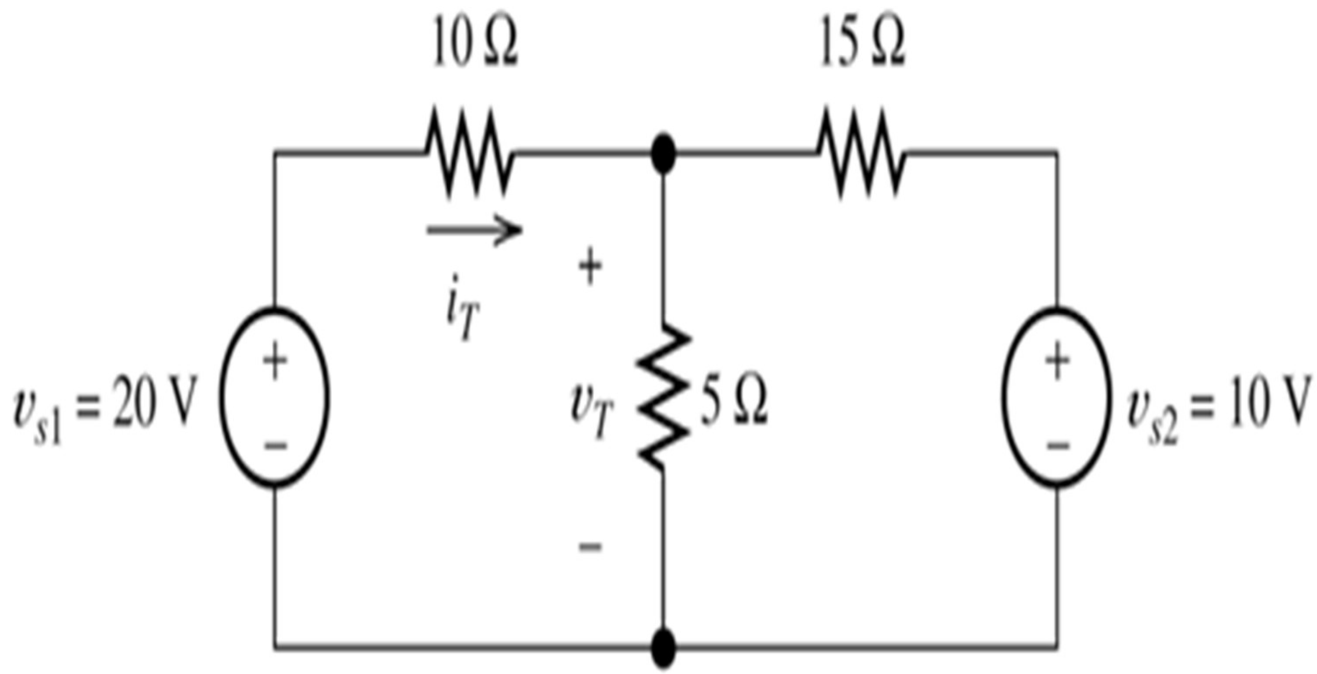


(c) Circuit with only the current source active

$$v_2 = i_s R_{eq} = i_s \frac{R_1 R_2}{R_1 + R_2} = (2A)\frac{(10)(5)}{10+5} = (2A)(3.33\Omega) = 6.66V$$

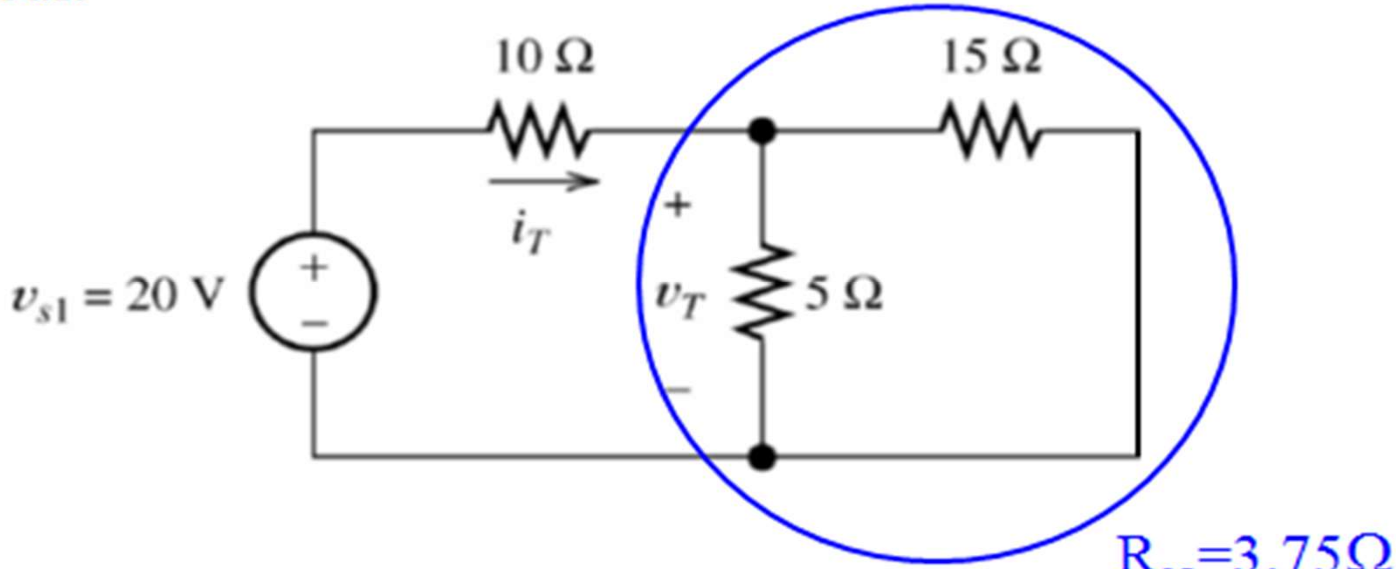
$$v_T = v_1 + v_2 = 5V + 6.66V = 11.66V$$

exam: use superposition find Vt



A - Circuit with just v_{s1}

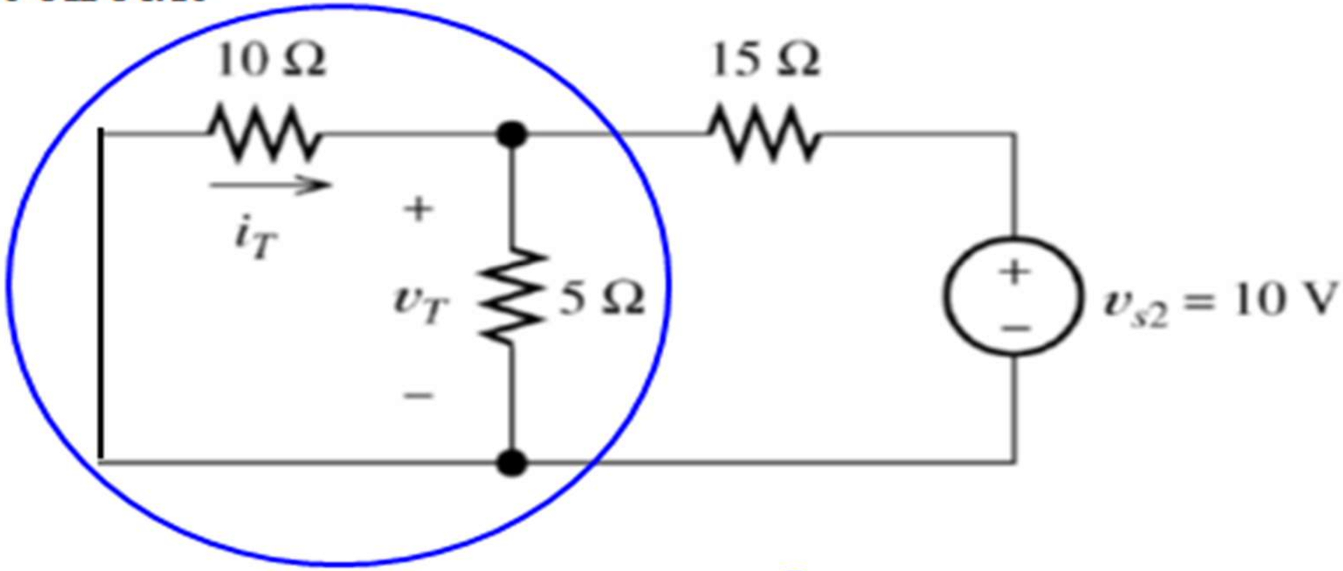
V_{s2} short circuit



$$v_{T1} = \frac{R_{eq}}{R_{eq}+10} v_{s1} = \frac{3.75}{3.75+10} 20 = 5.45V$$

B - Circuit with just v_{s2}

V_{s1} short circuit

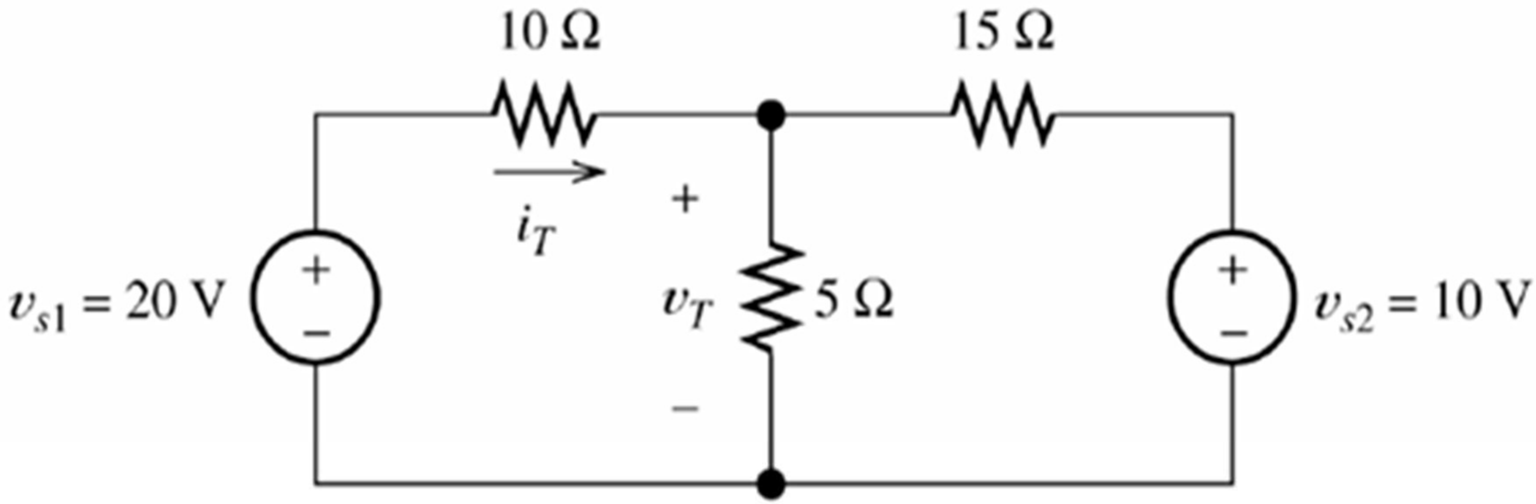


R_{eq}=3.33Ω

$$v_{T2} = \frac{R_{eq}}{R_{eq}+15} v_{s1} = \frac{3.33}{3.33+15} 10 = 1.82V$$

$$v_T = v_{T1} + v_{T2} = 5.45 + 1.82 = 7.27V$$

How to find iT

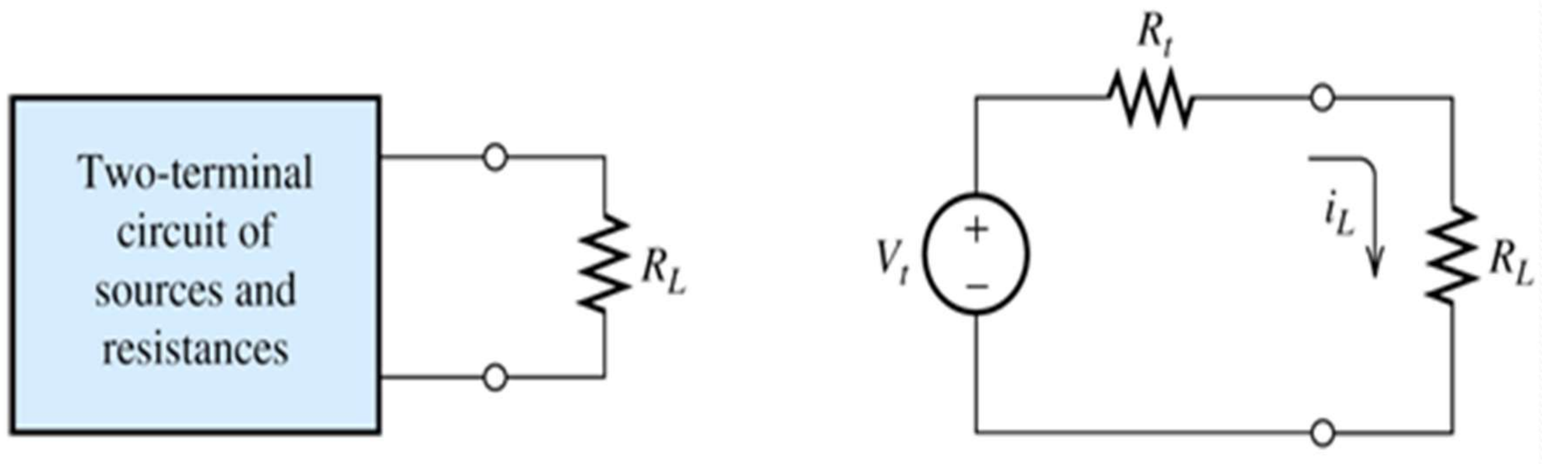


$$i_T = \frac{v_{s1} - v_T}{10\Omega} = \frac{20V - 7.27V}{10\Omega} = 1.27A$$

Maximum Power Transfer

- ***The load resistance that absorbs the maximum power from a two-terminal circuit is equal to the Thévenin resistance.***

Power will be max. when Load resistor is equal to thevenin,s resistance



(a) Original circuit with load

(b) Thévenin equivalent circuit with load

$$I_L = \frac{V_t}{R_t + R_L} \qquad P_L = i_L^2 R_L = \left(\frac{V_t}{R_t + R_L} \right)^2 R_L \qquad \frac{dP_L}{dR_L} = 0 \rightarrow R_L = R_t$$

Maximum Power Transfer Theorem

Proof:

$$P_L = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

Taking derivative w.r.to R_L we get,

$$\frac{dP_L}{dR_L} = V_{Th}^2 \{\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}$$

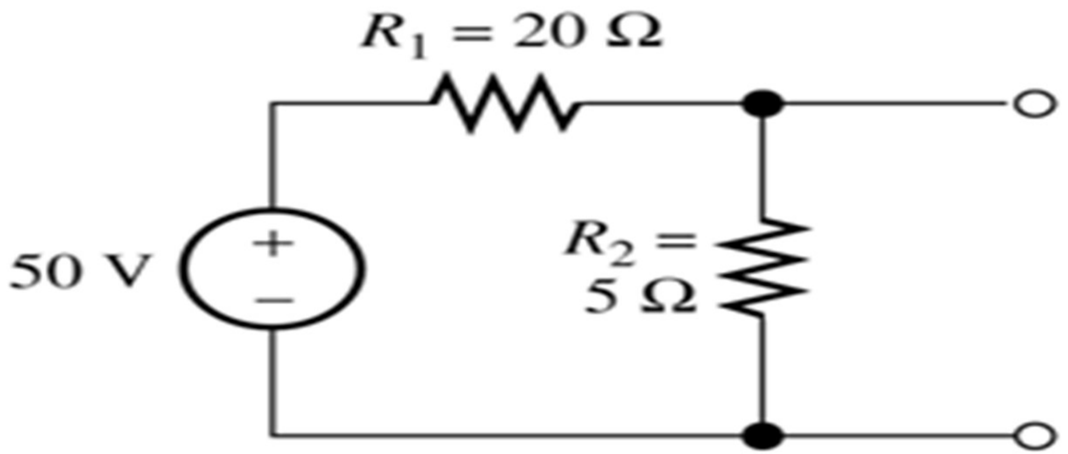
$$= V_{Th}^2 \{\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3} = 0$$

This imply that $(R_{Th} - R_L) = 0$

Therefore, $R_{Th} = R_L$

$$P_{max} = \frac{V^2{}_{Th}}{4R_{Th}}$$

Example : Find the load resistance for maximum power



$$R_t = \frac{R_1 R_2}{R_1 + R_2} = \frac{(20)(5)}{20+5} = 4\Omega \rightarrow R_{L_{Max}} = 4\Omega$$