Boolean Algebra and Simplification Techniques

DeMorgan’s Theorem

$$\overline{(x+y)} = \overline{x}\cdot\overline{y}$$
$$\overline{(x\cdot y)} = \overline{x}+\overline{y}$$

Example 1:- let’s apply them to the expression $\overline{(A\overline{B}+C)}$ as shown below:

$$\overline{(A\overline{B}+C)} = \overline{(A\overline{B})}\cdot\overline{C}$$

$$\overline{A\overline{B}}\cdot\overline{C} = (\overline{A}+\overline{\overline{B}})\cdot\overline{C}$$

$$(\overline{A}+B)\cdot\overline{C} = \overline{A}\overline{C} + B\overline{C}$$

Example 2:-

Simplify the expression $z = \overline{(\overline{A}+C)\cdot(B+\overline{D})}$ to one having only single variables inverted.

Solution

$$z = \overline{(\overline{A}+C)} + \overline{(B+\overline{D})}$$
$$z = \overline{(\overline{\overline{A}}+C)} + \overline{(B+\overline{\overline{D}})}$$
$$= (\overline{\overline{A}}\cdot\overline{C}) + \overline{B}\cdot\overline{\overline{D}}$$
$$z = A\overline{C} + \overline{B}D$$

H.W.

$$z = \overline{A+\overline{B}\cdot C} \qquad \omega = \overline{(A+BC)\cdot(D+EF)}$$

Rules of Boolean Algebra:-

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

Example 1: Using Boolean Algebra techniques, simplify the following expression:

$1 - X(\overline{X} + Y)$

$2 - AB(C + \overline{A}\,\overline{C})$

$3 - ABC + A\overline{B}C + AB\overline{C}$

Solution:-

$1 - X(\overline{X} + Y)$

$= X\overline{X} + XY$

$= XY$

$2 - AB(C + \overline{A}\,\overline{C})$

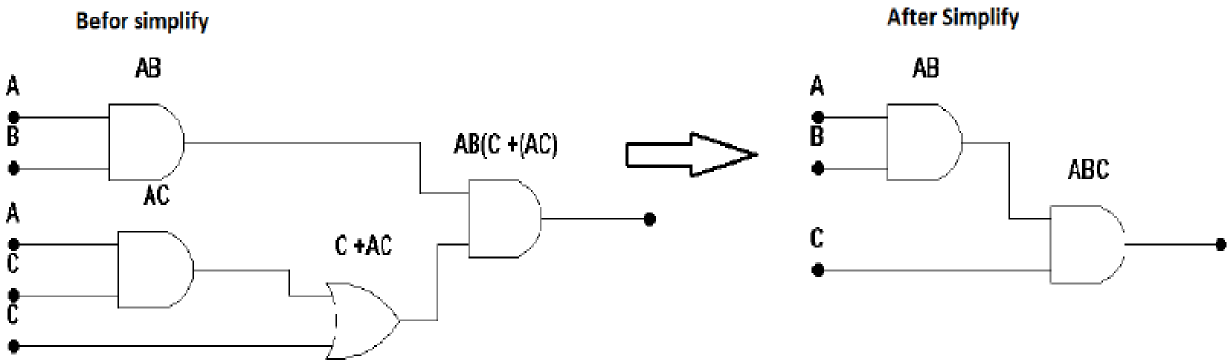
$= ABC + AB\overline{A}\,\overline{C}$

$= ABC + (A\overline{A})B\overline{C}$

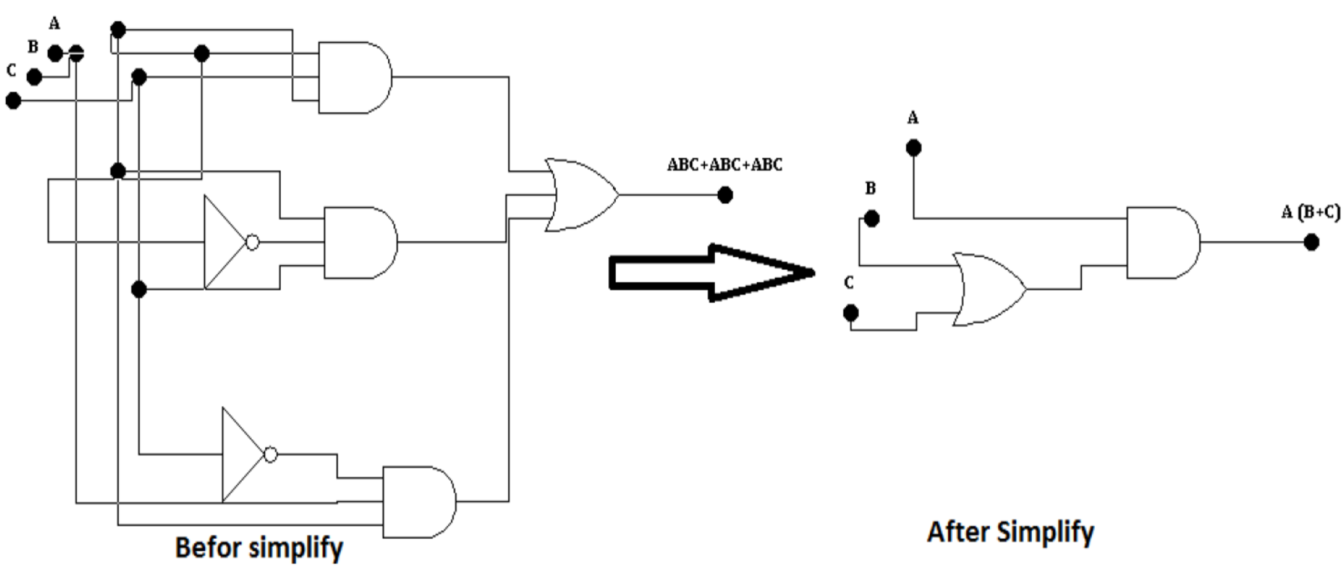
$= ABC + 0.B\overline{C}$

$= ABC + 0$

$= ABC$



$$3 - ABC + A\overline{B}C + AB\overline{C}$$
$$= AC\left(B + \overline{B}\right) + AB\overline{C}$$
$$= AC.1 + AB\overline{C}$$
$$= AC + AB\overline{C}$$
$$= A(C + B\overline{C})$$
$$= A(C + B)$$



H.W.

1 - ***Simplify the following expression and Draw the logic circuit before and after simplification***

a) $A\overline{B} + A\left(\overline{A+C}\right) + B\left(\overline{B+C}\right)$

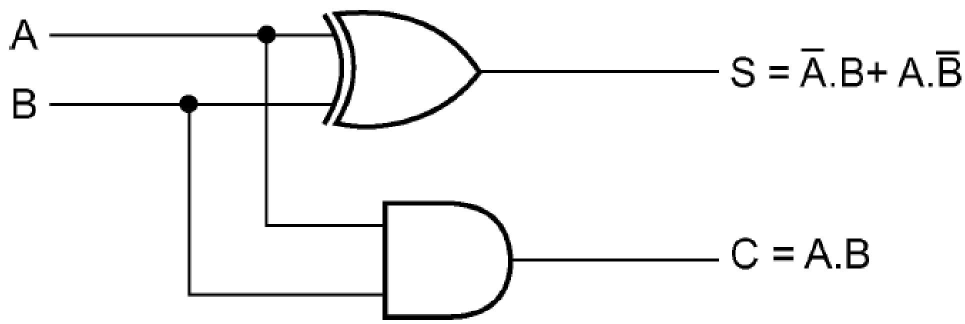
b) $\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$

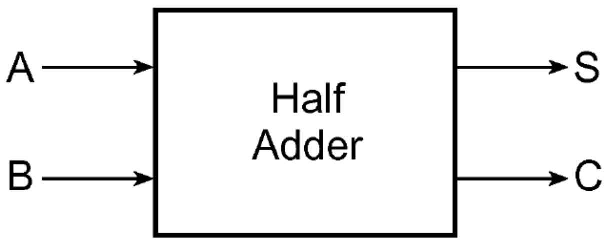
Arithmetic Circuits

Half-Adder

$$\text{SUM } S = A.\overline{B} + \overline{A}.B$$

$$\text{CARRY } C = A.B$$





A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Truth table of a half-adder

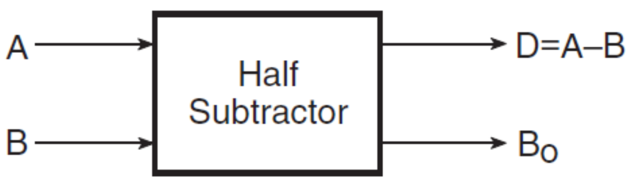
Half-Subtractor

$$D = \overline{A}.B + A.\overline{B}$$

$$B_o = \overline{A}.B$$

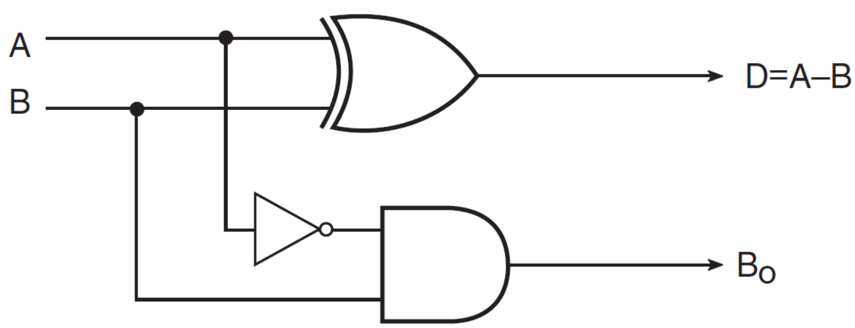
B_O = Borrow

D=Difference



A	B	D	B_O
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Logic diagram of a half-subtractor



Logic diagram of a half-subtractor